Math 421, Fourier Analysis

Suppose we sample a periodically varying quantity such as an intensity or a voltage. In the figure below we have sampled the tone produced by a button on a standard touch-tone telephone. Each tone consists of two frequencies: one of \( \{697, 770, 852, 941\} \) indicating the row and one of \( \{1209, 1336, 1447\} \) indicating the column.

Which button was pressed? A general method was found by Jean Baptiste Joseph Fourier (1768 - 1830) in 1807. His method is now known as Fourier analysis as is the subject of Math 421.

Fourier’s method was to write a given periodic function, \( f(t) \), with frequency \( \omega \), as a linear combination of \( \sin(2\pi k\omega t) \) and \( \cos(2\pi k\omega t) \). Fourier could check that his formulas worked for some functions but he claimed that his method worked for all functions. At this point there was not a generally agreed upon definition of a function, so his claim was controversial. Much mathematical work in the nineteenth and early twentieth centuries was devoted to providing a mathematical basis for Fourier’s claims.

Topics Covered:

- Discrete Fourier transform
- Ch. 1, Fourier series and the heat equation
- Ch. 2, Convergence of Fourier series
- Ch. 3, Sine and cosine series, arbitrary periods, \( \sin(x)/x \), Gibbs’s phenomenon
- Ch. 4, Convergence in \( L^2 \) and \( L^1 \) (time permitting)
- Fourier transform: transforms of translates, derivatives, convolutions, and multiplications by a polynomial; Riemann-Lebesgue Lemma (again), the Schwartz class, inversion theorem, Plancherel theorem, Poisson summation formula.

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Prerequisite: Math 328 (preferred), at least Math 281.

Text: *Fourier Series* by Rajendra Bhatia, and section 8.3 from *Real Analysis* by Gerald Folland.

Grading Scheme:

- five assignments 40%
- midterm examination 30%
- final examination 30%.

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