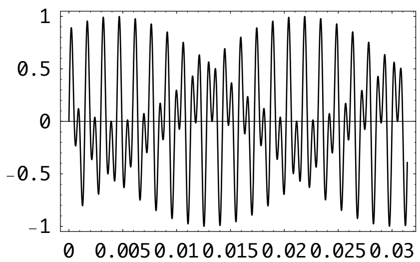
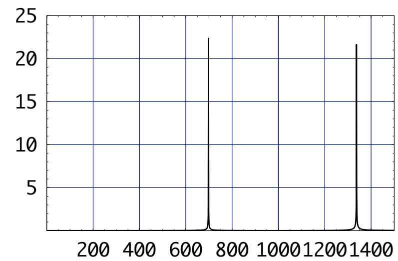
# Math 421, Fourier Analysis

Suppose we sample a periodically varying quantity such as an intensity or a voltage. In the figure below we have sampled the tone produced a button on a standard touch-tone telephone. Each tone consists of two frequencies: one of { 697, 770, 852, 941 } indicating the row and one of $\{1209, 1336, 1447\}$ indicating the column.




The sampled tone




The Fourier analysis of the tone

*Which button was pressed?* A general method was found by Jean Baptiste Joseph Fourier (1768 - 1830) in 1807. His method is now known as Fourier analysis as is the subject of Math 421.

Fourier's method was to write a given periodic function, $f(t)$, with frequency $\omega$, as a linear combination of $\sin(2\pi k\omega t)$ and $\cos(2\pi k\omega t)$. Fourier could check that his formulas worked for some functions but he claimed that his method worked for all functions. At this point there was not a generally agreed upon definition of a function, so his claim was controversial. Much mathematical work in the nineteenth and early twentieth centuries was devoted to providing a mathematical basis for Fourier's claims.

Topics Covered:

- Discrete Fourier transform
- Ch. 1, Fourier series and the heat equation
- Ch. 2, Convergence of Fourier series
- Ch. 3, Sine and cosine series, arbitrary periods, $\sin(x)/x$, Gibbs's phenomenon
- Ch. 4, Convergence in $L^2$ and $L^1$ (*time permitting*)
- Fourier transform: transforms of translates, derivatives, convolutions, and multiplications by a polynomial; Riemann-Lebesgue Lemma (again), the Schwartz class, inversion theorem, Plancherel theorem, Poisson summation formula.

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PREREQUISITE: Math 328 (preferred), at least Math 281.

TEXT: *Fourier Series* by Rajendra Bhatia, and section 8.3 from *Real Analysis* by Gerald Folland.

GRADING SCHEME:

five assignments 40%
midterm examination 30%
final examination 30%.

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