**MATH 386***

**Advanced Elementary Number Systems – Fall 2010**

This course is suitable for all students in a MATH Honours programme. It is one of the courses in the department's Teaching Focus.

**Instructor:** Prof. M. Orzech, 210 Jeffery Hall, 533-2436, orzechm@mast.queensu.ca

**Recommended background:** MATH 281*.

**Evaluation scheme:** 30% for homework; 25% for report and presentation on group-work project; 25% for final exam, 10% for in-class quizzes, 10% for instructional contributions to course (this 10% will be shifted to the final exam if this results in a better course mark).

**Homework:** There will be about 12 problems assigned over the term. You will generally have at least a week (and always a weekend) to work on each problem set.

**Course description:** Math 386 is about "number systems": integers, rationals, reals and extensions. We will examine some fundamental properties of real numbers and related number systems, and the connection of these properties to calculus and to other mathematical topics encountered earlier by students. We will discuss usual and unusual representations of numbers (e.g., in different bases, as sums of unit fractions, as continued fractions), which properties depend on representation, and why. The course touches in a "non-abstract" way on ideas from analysis. For example, convergence, upper bounds, and Cauchy sequence are introduced (reviewed for some students) in considering foundational properties of numbers that students often find difficult to explain or to illustrate or to connect to school-level mathematics they know. One goal is to probe the basis for our sense of familiarity with the real numbers and their arithmetic, including our understanding that the set of real numbers is complete. This will lead to the realization (echoing a $19^{th}$ century phenomenon) that using familiar convergence properties of real sequences can seem like "circular reasoning" until one explains how the real numbers arise from the rationals. The construction of real numbers as Dedekind cuts or as Cauchy sequences on the rationals will be outlined, with some exercises, but not complete proof. In past years there has been time for a segment on Liouville's construction of transcendental numbers.

Math 386 is part of the Teaching Focus of the math major programme. With this in mind, course material is intended not so much to extend the level of abstraction or generality of what students know from previous university courses as to connect that knowledge to what can be understood by high school students. The pedagogical complement of this will be an effort to promote student participation in discussing, presenting, and even selecting course material. Classes will feature a variety of activities, including lectures, class exercises and discussion, and student presentations (which may be with a partner, and for which guidance and preparation support will be available).

In addition, students will work (with a group) on an independent study. This work will be written up as a report, and each group will do a class presentation based on its work. Details about the report and presentation expectations will be provided later. Available topics (each meant for just one group) include:

*The development of logarithms and exponential functions*
*The development of complex numbers, quaternions and extensions*
*Replacing "epsilon-delta" arguments with infinitesimals*
*Comparing various measures of the "size" of infinite sets related to real numbers*
*P-adic numbers*