(——; 3-0-0) **Real Analysis** MATH-328*

Textbook: *Elementary Classical Analysis*, 2nd Edition
by J. E. Marsden and M. J. Hoffman (Freeman)

Prerequisite: MATH-281* *or* 220 *or* 223* (with permission of the instructor).

Instructor: L. B. Jonker

Evaluation:

Homework	70%
Final examination	30%

Outline:

The course covered roughly the first 7 chapters of the textbook.

From the **Introductory chapter** in the textbook: Sets and functions between sets, infinite sets and cardinality, countable sets; Schröder-Bernstein Theorem (not in our text); proof that the rationals are countable and that the interval [0,1] is not. (three lectures)

From **Chapter 1** in the textbook: completeness of the real line, density of the rationals in the set of reals, sup and inf, Bolzano-Weierstrass Theorem for the set of reals, cluster points (=limit points) of a sequence. (two lectures)

metric spaces, normed linear spaces and inner product spaces, non-standard examples of each, Cauchy-Schwarz inequality. (two lectures)

From **Chapter 2** in the textbook: topological spaces, examples of topological spaces, including subspace topology and pullback topology induced by a map into a topological space, open sets, interior of a set, closed sets, accumulation points, closure of a set, boundary of a set, and bf in the context of metric spaces: sequences, convergence, Cauchy sequences, completeness, brief discussion of series (on normed linear spaces), Banach spaces, absolute convergence, completeness of the space of continuous functions on a closed interval (uniform convergence). (three lectures)

From **Chapter 3** in the textbook: compact sets, sequential compactness, Bolzano-Weierstrass Theorem, totally bounded sets, Heine-Borel Theorem, Sketch of the proof of completeness of the space of compact subsets of a complete metric space with the Hausdorff metric between sets, nested set property, three definitions of continuity (from Ch. 4), path-connected sets, connected sets, limits. (four lectures)

From **Chapter 4** in the textbook: composition of continuous functions, continuous images of compact and connected sets, Maximum-Minimum Theorem, Intermediate Value Theorem, uniform continuity and the Uniform Continuity Theorem. (two lectures)

From **Chapter 5** in the textbook: uniform convergence, continuity of a uniform limit of a sequence or series of functions, Weierstrass M-test, the Cauchy Criterion and uniform Cauchy sequences, integration and differentiation of series, the space of bounded continuous maps as a metric space, and as a normed vector space if the image space is a normed vector space, with completeness if the image space is complete; equicontinuity and the Arzela-Ascoli Theorem; The Contraction Mapping Principle with applications to the existence theorem for ordinary differential equations, and to iterated function systems and the Collage Theorem (not in the textbook); the Stone-Weierstrass Theorem. (nine lectures)

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From **Chapter 6** in the textbook: Review of differentiable mappings between Euclidean spaces, the derivative as a linear map, calculation of the derivative matrix, especially for mappings on spaces of matrices, review of the chain rule and the product rule; the Mean Value Theorem; higher derivatives as multilinear maps; Taylor's Theorem. (five lectures)

From **Chapter 7** in the textbook: Inverse Function Theorem and Implicit Function Theorem, with applications. (five lectures)