(——; 3-0-0) **Variational Methods** MATH-432*

The calculus of variations provides the mathematical theory for optimizing a function which is defined on a family of curves. This function is usually a path integral. This theory has applications throughout every branch of scientific inquiry, including economics, physics, biology and of course mathematics. It also forms the basis for understanding properties of length minimizing curves on manifolds, and is used throughout physics under the rubric "principle of least action". We will give a modern treatment of this subject emphasizing recent applications such as the property of parametric resonance in physical systems, periodic trajectories for Hamiltonian systems of equations, and periodic orbits in N-body systems.

**Textbook:** *Assembled Notes*

**Prerequisite:** MATH-280* *and* 231* *and* 328*.

**Instructor:** D. Offin

**Outline:**

- Formulation of the problem and the different kinds of extremum
- First variation and the Euler Lagrange equations
- Corner conditions and non smooth extremals, strengthened Euler condition
- Smoothness of extremals, Hilberts theorem
- Invariance of the Euler equations
- Legendre transform and the Hamiltonian
- Variable endpoints and periodic boundary conditions
- Variational problems with geometric (holonomic) constraints
- Constrained mechanical systems, theory of oscillations
- Second variations and the Jacobi necessary condition
- Conjugate points and Morse index