The Diagnostic Tests

Students who have taken OAC Calculus normally take MATH 121 or MATH 126. If you think you may want to do a Mathematics or Statistics major or medial concentration, you should take MATH 120. If you have not had OAC Calculus or its equivalent the following diagnostic tests were design to help you decide which mathematics course you should take. There are two tests, Test 1 and Test 2. Roughly speaking, if you can't do well on Test 1, you prabably need to go back to high school or community college and sharpen your grade 12 skills; if you can do well on Test 1 but not on Test 2, you probably need to take MATH 006 before MATH 121. If you can do well on Test 2, go directly to MATH 121. We explain what "well" means below.

There is something to keep in mind. Mathematical skills don't have much staying power unless they're used, so it may be that you can see at glance that the questions on a certain test are questions you used to know how to do but have forgotten how to do in the past few months/years. In this case, what you might do is glance over the test and decide whether the problems seem familiar and whether you think a small review of certain skills might be in order. If so you might want to spend a few hours/days with a high school text brushing up the relevant sections before you try the test. That is, there's no point in committing yourself to an entire remedial course if all you need is a bit of a review. In short, the tests are designed to catch students either who have not had the material, or who have seen it but have never managed to get control over it.
 When you come to do the tests,

> You will need a supply of paper, a scientific calculator, a ruler and pencils.

The answers, marking scheme and decision criteria will be found on the following pages. Do **NOT** look at that material until you have completed, to the best of your ability, *both* tests.

Diagnostic Test 1

Simplify:

1.
$$\frac{(6y^3)^2}{2y^5}$$

2.
$$(12a^2bc)(\frac{8a^3b}{c^2})^{-2/3}$$

Factor completely:

3. $12x^3 + 3x$

4. $x^2 + 7x + 6$

5. $8x^3 + 14x^2 - 15x$

6. Simplify the expression:

$$\frac{(x+h)^3 - 7(x+h) - (x^3 - 7x)}{h}$$

Solve the following equations:

7.
$$\frac{2x-1}{x+3} = \frac{5}{6}$$

8. $x^2 - x - 12 = 0$

9. $2x^2 = 8x - 1$

10.
$$\frac{x+5}{x-1} = \frac{5}{x-7}$$

11.
$$\frac{1}{\frac{1}{x}+\frac{1}{4}} = 3$$

Write as a fraction with a common denominator:

12.
$$\frac{1}{ac} + \frac{1}{ab}$$

13.
$$\frac{5}{x^2-x-6} - \frac{3}{x^2+x-2}$$

Solve the simultaneous pairs of equations:

14.
$$\begin{aligned} x + y &= 37 \\ x - y &= 9 \end{aligned}$$

15.
$$\begin{aligned} 6x + 5y &= -11 \\ 10x &= 3(1-y) \end{aligned}$$

16. A right-angled triangle has sides of 7, 5 and x cms as shown. Find the value of x.

17. A kite is being flown over level ground on the end of a 150 m line. If the line makes an angle of 35 degrees with the ground, how high is the kite?

18. Find all the angles, $0^o \le \theta \le 360^o$, for which $\tan \theta = -1$.

19. Find the equation of the straight line of slope 2 through the point (5,-3).

20. Draw the graph of the equation

$$2x + 3y - 6 = 0$$

Diagnostic Test 2

Simplify:

1. $\log_2 12 + \log_2 20 - \log_2 15$

2.
$$2\log_5 2 + \log_5 15 - \frac{1}{2}\log_5 18 - \frac{1}{2}\log_5 8$$

Solve for x:

3. $\log_{10}(x+2) + \log_{10}(x-1) = 1$

4. $2^{x(x+3)} = 4^{5(x-1)}$

Simplify:

5. $\sin^2\theta + \cos^2\theta$

6.
$$\frac{\sin\theta}{\cos\theta}$$

7.
$$\frac{\tan\theta}{2} + \frac{\cot\theta}{2}$$

8. $\sin A \cos B + \cos A \sin B$

9. Expand $\cos(A+B)$

10. Solve for all θ in the domain $0 \le \theta \le 2\pi$
$$\sin^2\theta - \cos^2\theta + \sin\theta = 0$$

11. Find the radius and the coordinates of the center of the circle:
$$x^2 + y^2 - 10x + 4y + 20 = 0$$

12. With the dimensions given as shown, find x.

13. A tangent is drawn to a circle of diameter 4 from a pont P at distance 6 from the centre of the circle. What is the distance from P to the point of contact of the tangent? Draw a diagram.

14. If $z = (200)(3^t)$, find an expression for the variable t in terms of z.

15. An exponentially growing population ($P = P_o a^t$) now has 500 individuals. In 10 years time, the population will be 2,000. What will it be 15 years from now?

16. A radioactive substance is decaying according to the formula $m = m_o e^{-0.55t}$, where m is the mass remaining t years after starting with an original mass of m_o. Find the half life period (the time it takes for the mass of the substance to be halved).

17. Find the limit:
$$\lim_{n\to\infty}\frac{3x+4}{2x-1}$$

Differentiate with respect to x:

18. $\sin x$

19. e^x

20. $x^5 \sin x$

21.
$$\frac{e^x}{\sin x}$$

22. $\sqrt{2x^3 + 5x - 2}$

23. The value V of a puffball depends upon its radius r(cm) according to the formula
$$V = 3r^3 - 2r \qquad (r \ge 1)$$

At the moment r has value 2 cm and is increasing at the rate of 0.1 cm/h. At what rate is its value increasing?

24. The initial temperature of a heating panel is $20^o C$. The temperature increases at 4° per minute for 8 mins., then decreases at 2° per min. for 5 mins. What is its final temperature?

25. Find the maximum area of a rectangle drawn inside a semi-circle of diameter 20cm.

The Diagnosis

Answers to the test and advice on grading yourself are in the following two pages. If you had a mark of 80 or better on Test 1 and scored 70 or more on Test 2, then you will probably be able to cope quite comfortably with any of your first year Calculus courses (MATH 120, MATH 121, or MATH 126). If you scored less then 70 on Test 2, but scored 70 or more on Test 1, then you should consider taking MATH 006*. If your results were poor in both tests, then you will find it difficult to succeed in mathematics courses at university, and should consider upgrading your skills by taking high school courses up to the grade 12 level at least, either in high school or community college.

What should you do if you came close to scoring 70? Suppose you did well in Test 1, and scored, say 66 in Test 2. Here you could consider taking MATH 121 at the same time you take MATH 006* in the Fall term. Suppose you did poorly in Test 2, and scored, say 66 in Test 1. Here the decision must be yours, but if you do decide to take MATH 006*, you must understand that you will have to continually reach back and review high school mathematics during the course. You may wish to take MATH 006* in the Winter term so that you finish it closer to when you take the Calculus course you are preparing for.

We do not think it would be realistic to expect to succeed in MATH 006* or a 100 level Calculus course if your score for Test 1 is much below 65.

Answers Diagnostic Test 1

Each question is worth 5 marks. If you realize that you had the correct method and would have obtained the right answer except that you made an embarassingly silly mistake (like writing an exponent as 2, when it was given as 3) then deduct one mark. Be less lenient if you forgot a negative sighn in calculation: for instance, if you wrote $(3)(-2)=6$, then deduct 2 marks.

Questions 1 and 2 test your knowledge of the rules $a^m a^n = a^{m+n}, a^m/a^n = a^{m-n}$, and $(a^m)^n = a^{mn}$.

1. $18y$

2. $3b^{1/3}c^{7/3}$

Factoring

3. $3x(4x^2 + 1)$

4. $(x+6)(x+1)$

5. $x(4x-3)(2x+5)$
   If you only wrote $x(8x^2 + 14x - 15)$ and carried the solution no further, you only score one point.

Multiply out and simplify the numerator, then factor out the h.

6. $3x^2 + 3xh + h^2 - 7$

Equations

7. $x = 3$

8. $x = 4$, or $x = -3$

9.
$$x = \frac{8 \pm \sqrt{56}}{4} = \frac{4 \pm \sqrt{14}}{2}$$

10. $x = -3, 10$

11. $x = 12$

Simplifying fractions

12.
$$\frac{c+b}{abc}$$

13.
$$\frac{2}{(x-3)(x-1)}$$

If you only went as far as

$$\frac{2x+4}{(x-1)(x+2)(x-3)}$$

give yourself 4 marks.

14. $(x, y) = (23, 14)$

15. $(x, y) = (3/2, -4)$

16. $x = \sqrt{24}$ which can be written $2\sqrt{6}$

Trigonometry

17. 86 m

18. $\theta = 135^o$ or 315°. The calculator answer of -45° is not acceptable.

Coordinate Geometry

19. $2x - y - 13 = 0$ or $y = 2x - 13$ etc.

20.

Total your score, the result is out of 100.

$$\textbf{TOTAL} = \frac{}{100}$$

Answers Diagnostic Test 2

Each question is worth 4 marks. If you realize that you had the correct method and would have obtained the right answer except that you made an embarassingly silly mistake (like writing an exponent as 2, when it was given as 3) then deduct one mark. Be less lenient if you forgot a negative sighn in calculation: for instance, if you wrote (3)(-2)=6, then deduct 2 marks.

Logarithms and exponentials

1. $\log_2 16 = 4$

2. 1   If you left the answer as $\log_5 5$ lose one mark.

3. $x = 3$ is the only answer. If you also had $x = -4$ which lies outside the domain of the given logarithms, then lose one mark.

4. $x = 2$ or $x = 5$

Trigonometric identities

5. 1

6. $\tan \theta$

7. $\csc 2\theta = 1/\sin 2\theta = 1/2 \sin \theta \cos \theta$.

8. $\sin(A + B)$

9. $\cos A \cos B - \sin A \sin B$

10.
$$\theta = \frac{\pi}{6} \text{or} \theta = \frac{5\pi}{6} \text{or} \theta = \frac{3\pi}{2}$$
one mark for each of the first two answers, 2 marks for the right-hand answer.

11. Center$= (5, -2)$ Radius$= 3$

12. $x = 96$

13.
$$x = \sqrt{32} = 4\sqrt{2}$$

14.
$$t = \frac{\log_a(z/200)}{\log_a 3} \text{where} \quad a \text{ is any base.}$$

$$\text{e.g.,} t = \frac{\ln z - \ln A}{\ln c} \quad \text{or} \quad t = \log_3(z/200)$$

15. $4,000$

16. 12.6 years

17. $3/2$

18. $\cos x$

19. e^x

20. $5x^4 \sin x + x^5 \cos x$

21.
$$\frac{e^x \sin x - e^x \cos x}{\sin^2 x}$$

22.
$$\frac{6x^2 + 5}{2\sqrt{2x^3 + 5x - 2}}$$

Related rates problem

23. 3.4

24. 42^oC.

25. Introduce a variable. Find the area of the rectangle in terms of that variable. Put the derivative of the area with respect to the variable equal to zero etc. Area$= 100$ cm^2

Total your score, the result is out of 100.

$$\mathbf{TOTAL} = \frac{}{100}$$