Queen's University at Kingston

Department of Mathematics and Statistics

Information on 300- and 400-level courses to be taught in 2007/08

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Some important things to note:

1. Students who would like advice about courses or programs should contact Professor Nielsen by

   - drop-in: ask for him at the mathematics and statistics office or
   - appointment: make an appointment at the mathematics and statistics office or by phone at 613-533-2390 or
   - e-mail: nielseno@post.queensu.ca.

2. STAT 251* and 261* will not be taught in 2007/08 or thereafter. These two courses have been replaced by STAT 268* and 269*. Students who have taken STAT 263* will not be allowed to take STAT 268* but may take STAT 269*; taking STAT 263* and 269* will satisfy the second-year course requirement in statistics and probability in all degree programs.

3. The courses that we are planning to teach in the 2007/08 academic year and the instructors for some of them are listed on page 3. Detailed descriptions of the contents of some of these courses is on pages 5-9. Please keep in mind that

   - the instructors may change and
   - courses with low pre-registration enrollments may be canceled.

4. Students doing a major in mathematics are required to select their 300- and 400-level courses in accordance with one of the foci listed in the calendar. For convenience, these foci as they will appear in the 2007/08 calendar are listed on page 4.

5. Some courses are offered in alternate years. The following courses WILL be offered in 2007/08 and will probably not be offered in 2008/09:

   Math 314*, 402*, 405*, 418*, 432*, 436*, STAT 462*, 484*.

   And, correspondingly, the following courses will NOT be offered in 2007/08 but will probably be offered in 2008/09:

   MATH 341*, 382*, 427*, 443*, 481*, STAT 460*, 466*.

6. I do not know if we will be teaching STAT 486* next year.

300- and 400-level courses to be taught 2007/08:

Number	Course title	Term	Instructor
MATH 310	Group Theory	F	Geramita
MATH 326	Functions of a Complex Variable	F	Bogoyavlenskji
MATH 334	Methods of Applied Mathematics I	F	Lewis
MATH 337	Introduction to Operations Research Models	F	
MATH 338	Topics in Applied Mathematics	F	
MATH 381	Mathematics with a Historical Perspective	F	Orzech
MATH 384	Mathematical Theory of Interest	F	
STAT 351	Probability I	F	Linder
STAT 361	Applied Methods in Statistics I	F	Levit
BIOM 300	Modeling Techniques in Biology	W	
MATH 311	Elementary Number Theory	W	
MATH 312	Linear Algebra	W	
MATH 314	Introduction to Galois Theory	W	Yui
MATH 328	Real Analysis	W	Nielsen
MATH 385	Life Contingencies	W	
MATH 387	Elementary Geometry - An Advanced Perspective	W	Orzech
STAT 353	Probability II	W	Takahara
STAT 363	Fundamentals of Statistical Inference	W	
MATH 401	Graph Theory	F	Murty
MATH 405	Applications of Matrix Algebra	F	Rice
MATH 474	Information Theory	F	Alajaji
STAT 455	Stochastic Processes and Applications	F	Takahara
STAT 462	Computational Data Analysis	F	
STAT 464	Discrete Time Series Analysis	F	
MATH 402	Combinatorics: Enumeration and Design	W	
MATH 406	Introduction of Coding Theory	W	Kashyap
MATH 418	Number Theory and Cryptography	W	Kani
MATH 432	Variational Methods	W	Offin
MATH 436	Partial Differential Equations	W	Bogoyavlenskji
MATH 439	Lagrangian Mechanics, Dynamics, and Control	W	Lewis
STAT 465	Quality Management	W	
STAT 484	Telecommunications and Data Network Modeling	W	Takahara
STAT 486	Survival Analysis	?	

Foci for students doing a major in mathematics:

The program for Years 3 and 4 must include a focused group of courses chosen from one of the following lists. In planning their program students should consider that many of the 400-level courses listed are not offered every year, but can be taken in third year. Graduate courses such as MATH 844*, 891*, 892*, 894*, and STAT 895* and 896* may be taken by students with an excellent record who obtain permission of the instructor and the undergraduate chair.

ACTUARIAL FOCUS: MATH 384*, 385*; STAT 353*, 361*; at least 2.0 credits chosen from MATH 337*, 434*, STAT 363*, 455*, 462*, 464*, 465*; an additional 1.5 credits chosen from COMM 211* (or 111*), 221* (or 121*), ECON 110, 212*, MATH 272*.

BIOMATHEMATICS FOCUS: BIOM 300*; at least 1.0 credit chosen from MATH 337*, 427*, 432*, 434*, STAT 455*.

BUSINESS FOCUS: MATH 337*, 384*; STAT 361* and/or 363*; at least 1.0 credit chosen from MATH 401*, 434*, STAT 353*, 455*, 464*, 465*.

COMMUNICATIONS FOCUS: MATH 312*, 328*; STAT 455*; at least 1.5 credits chosen from MATH 406*, 418*, 474*, 477* and 484*.

DISCRETE MATHEMATICS AND OPTIMIZATION FOCUS: MATH 312*; at least 2.0 credits chosen from STAT 353*, MATH 401*, 402*, 405*, 406*, 434*.

DYNAMIC PROCESSES FOCUS: MATH 326*, 328*; at least 1.0 credit chosen from STAT 353*, MATH 427*, 432*, 441*.

PROBABILITY FOCUS: MATH 328*; STAT 353*, 363*, 455*; at least one of MATH 474*, 484*.

PURE MATHEMATICS FOCUS: 2.5 credits from MATH 312*, 310* (or 313*), 314*, 326*, 328*, 341*.

STATISTICS FOCUS: STAT 361*, 363*; at least 1.0 credit chosen from STAT 460*, 462*, 464*, 465*, 466*, 486*.

TEACHING FOCUS: At least 1.5 credits chosen from MATH 311*, 313*, 381*, 382*, 386*, 387*.

Math 381* and Math 387*

The structure of these courses recognizes that they are part of the department's Teaching Focus. The requirement of a group project is one aspect of this recognition. In Math 381* the project will lead to a major written report. In Math 387* it will culminate in a fifty-minute presentation to the class, plus a written summary. The project in both courses will involve exploring the historical, mathematical, and pedagogical development of a suitable topic. There will be guidance and direction, but also choice, in selecting a project topic. There are other aspects to the Teaching Focus orientation of Math 381* and Math 387*. Class activities and student involvement will complement the lecture-based presentations. Pedagogical issues will sometimes be made explicit, particularly as they relate to historical and pedagogical development of mathematics. In Math 381* about one class a week will be devoted to short student presentations (with a partner) about mathematical issues that arise from the reading (often involving a high school topic viewed with the benefit of university mathematics experience).

Math 381* Mathematics with a Historical Perspective

The course progress will be guided by the text, *Journey Through Genius* by W. Dunham. The book has twelve chapters. The n-th week of classes will generally be based on the n-th chapter, but class activities and presentations will introduce other material connected mathematically or historically to Dunhams exposition. Course prerequisites include single-variable differential and integral calculus, linear algebra, and the ability to recall high school mathematics with ease. Calculus will be called upon more than linear algebra. The latter prerequisite is there to ensure an interest in mathematics, and prior exposure to mathematical areas and modes of thought consistent with the mathematical scope and intellectual character of a third year mathematics course. For example, proof will be an important element of the course, as will a range of mathematical knowledge sufficient to make some of the general and abstract perspectives encountered personally meaningful. Since Dunham's book is written for a non-specialists, a good deal of the course work will depend on high-school mathematics, and you will need to access that material comfortably, and to interpret it with a more sophisticated and flexible perspective than in high school.

Math 387* Elementary Geometry, an Advanced Perspective

This course is an in-depth follow-up to high school geometry: The course will present striking geometric results or proofs (modern and traditional, and likely new to you) about plane and solid figures (e.g. polygons, polyhedra, circles, spheres, conic sections). Connection between results, and with other mathematical subjects, will be elucidated. The course will present alternative approaches (e.g. traditional synthetic methods, vector-based and transformation-based methods) to proof and interpretation of geometric results. Extensions from Euclidean plane geometry to solid, spherical, hyperbolic, and projective geometry will be examined. It will be proved, using modern algebra, why the classical Greek construction problems are unsolvable using straightedge and compass. Physical models, and technology such as Geometers Sketchpad, will be used for geometric exploration.

MATH 405* Matrix Theory and Applications

Matrices turn up in an astonishing variety of mathematical situations. This course looks at a sampling of some of the techniques and applications that I find particularly interesting. Part of the good news is that these topics are not only very interesting and useful, they are also quite concrete with only a modest technical background required, so the course is quite accessible to third year students.

Here are a few of the kinds of questions we will look at:

- To solve a system of linear equations y = Ax you might invert A and multiply, but this is hardly ever the best thing to do. So what is better? And what if A is not invertible but you need some sort of approximate solution anyway? Or what if there are many solutions, but you need some smallest solution? or one in integers?
- We all know what A^2 means for a matrix A, but what does e^{At} mean? and how can we calculate it?, and why would we want to?
- It turns out for the previous problem that we need to estimate some eigenvalues. But what is a reasonable way to do this? (No, it is not by solving the characteristic equation.)
- Suppose we have a very large matrix that we want to approximate with a matrix of low rank. Why? How? (And how did Prof. David Scillicorn get famous doing this to Enron emails?)

Math 418

Number Theory and Cryptography

Winter 2008

Cryptography, the art of writing secret messages, is nowadays an essential ingredient of any communication system that handles sensitive information. Although cryptography has been practiced for several milleniums, its large scale use became only practical with the invention of Public Key Cryptography in 1976. The latter is based on results and techniques in elementary Number Theory and Algebra is the main focus of this course.

Course Outline:

- **Time estimates:** Polynomial time algorithms, time estimates for arithmetic algorithms (division algorithm, Euclidean algorithm, congruences), modular arithmetic, fast exponentiation, solving $x^n \equiv 1 \pmod{m}$, extracting square roots $\pmod{p}$ (using quadratic residues); computations in finite fields.
- **Cryptography:** Classical and Public Key Cryptosystems (RSA, discrete logarithm). Attacks on crtosystems (Pollard's rho method, index calculus); primality tests
- **Elliptic Curve Cryptography:** Elliptic curves (basic properties), elliptic curve cryptosystems. Applications of elliptic curves to (i) primality tests and (ii) factorization of integers.

Textbook: N. Koblitz, *A Course in Number Theory and Cryptography* 2nd edition. Springer-Verlag, 1994.

Prerequisite: MATH 210* or 212* or 217*; or MATH 211 with permission of the Department.

MATH 432* Calculus of Variations

The Calculus of Variations is a field of research approximately 300 years old. Newton anonymously solved the famous Brachistochrone problem (see below) set by Johann Bernoulli. Since those beginnings, the calculus of variations has grown and developed to the point where it may be said to supply one of three pillars of the modern theory of mathematical analysis. The other two being the differential geometry of real and complex manifolds, and classical and modern mathematical physics. Needless to say, these three fields are not disjoint, and there are many interweaving threads throughout these topics not least of which is a deep source of nonlinear phenomenon throughout science and engineering. As an example, almost all of classical mechanics of particles under conservative forces can be cast in the language of variational problems. The basic problem we wish to consider is one analogous to the familiar problem of choosing the shortest path between two points, from elementary geometry. This generalization first assigns to each parameterized curve in some region of the plane or higher dimensional Euclidean space, a performance value, similar to the arclength of the curve but perhaps using some other measurement. Then the following questions naturally arise in this investigation. Among the curves so considered, are there optimal ones, which either minimize or maximize the performance value relative to other curves under consideration. How would one recognize such an optimal curve, are there necessary or sufficient conditions which can be applied to determine whether we have a good candidate for the best curve in the class. If there is no optimal curve in this sense, can we enlarge the class of curves under consideration so as to provide a positive solution to the problem of minimization or maximization? Are we always guaranteed of finding such an optimal curve? For nonlinear problems, are there reasonable techniques which can be used to isolate the best candidates for optimality? This brief list of possible questions is not complete and the answer to many of them in general depend on the specific variational problem under consideration. Some of the classical problems in the calculus of variations which can be analyzed are 1. shortest paths on polyhedral, cylindrical, conic or spherical surfaces. 2. The more general problem of minimum arclength for smooth curves on a surface or manifold of Euclidean space . 3. The brachistochrone problem in the plane, that is the curve upon which a mass m falling under constant gravitational acceleration can transit between two points in the plane, in minimum time. 4. The isoperimetric problem, a generalization of the ancient problem of enclosing the maximum area with a curve of given arc length. 5. Minimum surface area for a surface of revolution, obtained when the graph of an arbitrary function on an interval I is rotated around the x-axis. 6. The hanging cable problem which determines the shape of the curve which assumes the least gravitational potential energy over the class of curves with given arc length between two points in the plane. This short list is merely representative of the kind of problems we shall consider amongst others. Our goal is to provide an introduction to this vast topic which can reasonably be expected to provide the basis for graduate research. Prerequisites: Math 231, Math 280/281, Math328 (optional but recommended).

Math 436* Partial Differential Equations

Text: *Partial Differential Equations* by by R. C. McOwen (Prentice Hall)

Prerequisites: MATH 231* or 237*, 280*

Recommended: One of MATH 328* or 334* or 338*, PHYS 312*

Outline:

- Cauchy problem for quasilinear equations
- Method of characteristics; Conservation laws and jump conditions
- Higher-Order equations
- Dispersion relations; Dissipation
- Power series and the Cauchy-Kovalevski theorem
- Wave equation; Spherical means
- Huygen's principle
- Conservation of energy; Domain of dependence
- Laplace equation
- Mean value and the maximum principle
- Boundary value problems
- Potential theory; Green's functions
- Heat equation
- The maximum principle; Uniqueness
- Cole-Hopf transform; Burgers' equation
- Korteweg - de Vries equation; Soliton solutions