Let a group \( G \) be non-commutative but all its subgroups \( H \) are commutative.

Let \( H \) be a subgroup of \( G \) and let \( a \) be a generator of \( G \) which is not a generator of \( H \) and not in \( H \). Let \( b \) be an element of \( H \), perhaps a generator of \( H \). In \( G \), \( ab = ba \).

Let \( a^n b = b a^n \) but also

\[
a^n b = a^{-1} ab = a^{-1} bac = a^{-2} ab ac = a^{-3} b ac^2 = \ldots = b(ac)^n.
\]

So \( b a^n = b(ac)^n \). So

\[
a^n = (ac)^n.
\]

Take \( n \)th roots, \( a = ac \) if \( a^n \) is not in a relation in \( G \). So \( c = 1 \). So \( ab = ba \) in \( G \).

So \( G \) is also commutative.