Problems 1a, p. 86, K. H. Hofmann and J. M. Mostert "Problems about compact semigroups" (pp. 85-123) in (ed.) K. W. Fulks "Symposium on Semigroups Held at Wayne State University, Detroit, Michigan, 27-29 June, 1968"

Let $S$ be compact connected, Hausdorff topological semigroup with zero, identity and group $T$ of units, let $T$ be the centralizer of $T$. Is $T$ connected?

But let $T$ be quaternions of length 1 and let $S = \mathbb{H} \times \mathbb{R}/\mathbb{H} \times \{0\}$. I am an $\mathbb{R}$-semigroup. Then $S = \{1\} \times (\mathbb{R} \setminus \{0\})$ not connected.

A positive answer would have deepened insight into structure of $S$ near identity, but the answer is negative.

In our example, $T$ is compact connected and $S$ is non-commutative, as is $T$, using $T = \{1, -1, i, -i, j, -j, k, -k\}$ of the quaternions if we get a counterexample with $T$ the same disconnected set.

"On problems of Hofmann and Mostert on the centralizer of a continuum semigroup."
A case with $S$ and $H$ commutative is given by $S = (C \times I) / (C \times \{0\})$, where $I$ is an $(\mathbb{R}, \cdot)$-semigroup and $C = \left\{ e^{\frac{2\pi i t}{n}} \right\}$, $t = 0, \ldots, 2\pi \mathbb{Z} / (n-1)$, $n > 1$, and integral.

Vaguely, I seem to remember thinking of this counterexample about 1/2 years ago (or further back into the past).