Catalan conjecture: \( x^u - y^v = 1 \), where \( x, y, u, v \) are integers > 1. Is \( 3^2 - 2^3 = 1 \).

gcd(x, y) = 1 is needed.

Maybe \( y \) even, \( x \) odd. If otherwise, it is not both odd; it is not both even, these restraints being to bring difference of powers to be odd.

Also, if \( x \) even, \( y \) odd, then \( y = (2a + 1) \) an integer; and \( y^v \equiv (1 + 2av) \pmod{4} \).

For this to satisfy the equation, \( v \) is odd if \( y \) is odd (and \( x \) is even).

\[
(x-y)(x^{u-1} + x^{u-2}y + \ldots + y^{u-1}) + y^{\min(u, v)} \left( y^{v \cdot \min(u, v)} - y^{v - \min(u, v)} \right) = 1.
\]

These seem difficult to satisfy.

Modulo \( y^2 \), \( (x-y)(x^{u-1}) \equiv 1 \pmod{y^2} \).

Gcd(x, y) = 1, so this is \( (x^2 y^2) \equiv 1 \pmod{y^2} \).

So \( (x^2 - y^2 - 1) y^2 = 1 \); namely,

Rell's equation \( x^u - Ny^v = 1 \). So \( x^u y^v = 1 \) has exactly one solution.