Q.

AN ELEMENTARY PROOF OF BOTH CASES OF FERMAT'S LAST THEOREM.

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Wiles proved it with an advanced proof. See:

Wiles
Taylor-Wiles

Annals of Math.

books on it:
van der Poorten
Ram Murty
Kumar Murty

Etc.
FLT proved for exponent 4 by Fermat: proof extant. Fermat claimed to have proved that
\[ x^n + y^n = z^n \]
has no solutions in non-zero integers \( x,y,z \) for each integer exponent \( n > 2 \). His proof is lost.

FLT fails when
We see that FLT holds for exponent \( mn \) if it holds for exponent \( n, m > 1 \) and integral.
So to complete it would suffice to prove it for all odd prime exponents \( p \).

FLT fails for odd prime exponent \( p \) if there exist relatively prime integers \( x,y,z \) such that \( p \) does not divide \( xyz \) but \( x^p + y^p = z^p \).

FLT fails for odd prime exponent such that \( p \) divides \( xyz \) but \( x^p + y^p + z^p = 0 \).
Consider FLT 1 for \( p > 5 \).

Let rel prime integers \( x, y, z \) exist such that \( p \) does not divide \( xyz \) but \( x^p + y^p + z^p = 0 \). We obtain a contradiction, proving FLT 1.

\[
(y + z) \text{ and } \left( \frac{y^p + z^p}{y + z} \right) \text{ have gcd 1, so both are } p \text{th powers.}
\]

Let \( y + z = a^p \) with \( a \mid x \)

\[
3 + x = b^p \text{ with } b \mid y
\]

and \( x + y = c^p \) with \( c \mid z \).

\[
\begin{align*}
2x &= b^p + c^p - a^p \\
2y &= c^p + a^p - b^p \\
2z &= a^p + b^p - c^p
\end{align*}
\]

so

\[
(b^p + c^p - a^p)^p + (c^p + a^p - b^p)^p + (a^p + b^p - c^p)^p = 0 \quad \text{(1)}
\]
One of \( x, y, z \) is even. Let \( y, z \) be odd. Then \( a^k \) is even; \( a \) is even. Let \( 2^k \mid x \), \( 2^{(k+1)} \mid x \) \( \forall k \geq 1 \) and integral. Then \( 2^k \mid a^k \), \( 2^{(k+1)} \mid a \).

Look at (1). Each term is in

\[
\frac{p!}{u!v!w!} (a^u)(b^v)(c^w)
\]

where \( u, v, w \) are integers \( \geq 0 \) and \( u + v + w = p \).

If \( u, v, w \) are all odd, then we get

\[
-3 \frac{p!}{u!v!w!} (a^u)(b^v)(c^w)
\]

Take this to the other side and add terms to make

\[3(a^l + b^l + c^l)\]

We get
$$3(a^2+b^2+c^2)^p = 4(a^2+b^2)^p - 4c^{2p}$$
$$+ 4(b^2+c^2)^p - 4a^{2p}$$
$$+ 4(c^2+a^2)^p - 4b^{2p}$$
$$+ 2a^{p}\, \text{and terms in } a^{2p}$$

(a)

$$a^2+b^2+c^2 = 2(x^2+y^2+z^2), \text{ so}$$

$$2^{p+1} \left| \frac{3}{2} (a^2+b^2+c^2)^p \right.$$ 

- $$2^{p+1} \mid \text{Each term in } a^{2p}$$
- $$2^{p+1} \mid 2a^p (b^2+c^2)^{p-1}$$
- $$2^{p+1} \mid 2a^p (b^2+c^2)^{p-1}$$

Modulo $$2^{p+5}$$ (a) is

$$0 \equiv 4(a^2+b^2)^p - 4b^{2p}$$
$$+ 4(c^2+a^2)^p - 4c^{2p}$$
$$+ 4(b^2+c^2)^p - 4a^{2p}$$.
This simplifies to
\[ 0 = 4aP \{ (b^p)^{p-1} + (c^p)^{p-1} \} + 4(b^p + c^p) \] —— (3)
but \( b^p + c^p = y + z + 2x \equiv 0 \pmod{2^{p+1}} \).

(3) is thus
\[ 0 = 4aP \{ (b^p)^{p-1} + (c^p)^{p-1} \} \pmod{2^{p+5}} \]
Divide by \( 4aP \equiv 0 \pmod{2^{p+2}} \)
\[ 0 \equiv \{ (b^p)^{p-1} + (c^p)^{p-1} \} \pmod{2^3} \]
but \( b + c \) are odd and \( (p-1) \) is even power. So \( (b^p)^{p-1} \equiv 1 \pmod{8} \), etc.

So \( 0 \equiv 2 \pmod{8} \),
This contradiction proves FLT 1.

Time is too short to give special proof for \( p = 3 \) differences.

I think that Fermat had this proof.