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FLT: IIM XMM.

Let p be an odd prime and let x, y, z be relatively prime integers such that $p \mid z$ and $x^p + y^p + z^p = 0$.

Let $k \geq 1$ be integral such that $p^k \mid z$ but $p^{k+1} \nmid z$. Let $x+y = cz^{p^{k-1}}$ for some $c \in \mathbb{Z}$.

Consider $x^{pk} - y^{pk}$

$$\begin{aligned} x^{pk} - y^{pk} &= (x^p - y^p)z^{p(k-1)} - (x^p - y^p)z^{p(k-1)} + \dots \\ &= (x^p + y^p)z^{p(k-1)} - (y^p + z^p)z^{p(k-1)} \\ &= (x^{pk} - y^{pk}) + \binom{p}{1} z^p (x^{p(p-1)} - y^{p(p-1)}) \\ &\quad + \binom{p}{2} z^{2p} (x^{p(p-2)} - y^{p(p-2)}) \\ &\quad + \dots \end{aligned}$$

So $0 = p z^p (x^{p(p-1)} - y^{p(p-1)}) + \frac{p(p-1)}{2} z^{2p} (x^{p(p-2)} - y^{p(p-2)}) + \dots$

$$\begin{aligned} 0 &= p(x+y)z^p \left[(x+y) \binom{p-1}{1} - p \binom{p-1}{1} y^{p(p-1)-1} \right] \\ &\quad + \frac{p(p-1)}{2} z^{2p} \left[(x+y) \binom{p-1}{2} z^{p(p-2)} - y^{p(p-2)} \right] \\ &\quad + \dots \end{aligned}$$

~~xxxxxx~~ $\times \mathcal{I}^p$ (2)

$$0 = -z^{2p} [(x+y)(\dots) - p(p-1)y^{p(p-1)-1}] \\ + z^{2p} \frac{p(p-1)}{2} [(x+y)(\dots) - 2y^{p(p-2)}]$$

Modulo $c \frac{3p(3p-1)}{p}$

$$0 \equiv + z^{2p} p(p-1) y^{p(p-1)-1} \\ - z^{2p} \frac{p(p-1)}{2} \mathcal{I}^p y^{p(p-2)}$$

$$\div p z^{2p} (p-1) y^{p(p-2)}$$

$$0 \equiv y^{(p)} + \mathcal{I}^p \pmod{c \cdot p^{(p-2)}}$$

$$\mathcal{I}^p \equiv y^{(p)} \pmod{c \cdot p^{(p-2)}}$$

This disagrees with $\mathcal{I}^p \equiv y^{(p-1)} \pmod{(x+y)}$

$\equiv y^{(p)} \pmod{c \cdot p^{(p-1)}}$
Go backwards and get a contradiction?