① ,

FLT1+2.

~~Let~~ p be an odd prime and ~~let~~ $p \nmid xy$ and ~~let~~ $x^p+y^p+z^p=0$. Maybe $p \mid z$.

Then $x+y=c^p$, etc; $x=-a\xi$, etc.

Now $(x+y+z)(xy+yz+zx)$

$$= x^2(y+z)+xyz+y^2(z+x)+xyz$$
$$+z^2(x+y)+xyz$$
$$= x^2(y+z)+y^2(z+x)+z^2(x+y)+3xyz.$$

But $(x+y+z) \equiv 0 \pmod{abc}$.

So $0 \equiv \xi^2 a^{(p+2)} + \eta^2 b^{p+2} \pmod{c}$. ......(1)

Also $\xi a + \eta b \equiv 0 \pmod{c^p}$ ————(2)

(2) gives $\xi^2 a^2 \equiv \eta^2 b^2 \pmod{c^p}$ ————(3)

~~Put~~ (3) in (1):

$$0 \equiv \eta^2 b^2 a^p + \eta^2 b^{(p+2)} \pmod{c}.$$

So $a^p + b^p \equiv 0 \pmod{c}$. Known!

Also, $(x+y+z)(xy+yz+zx) =$

$$x(y^2+z^2)+y(z^2+x^2)+z(x^2+y^2)+3xyz$$

So $0 \equiv xy^2+yx^2 \underset{(\mathrm{mod}\ c)}{\equiv} xy(y+x) \equiv 0 \pmod{c^p}$.

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But $(x+y+z)(xy+yz+zx)$

$= x(y+z)^2 + y(z+x)^2 + z(x+y)^2 - 3xyz$

So $0 \equiv \xi a^{(2p+1)} + \eta b^{(2p+1)} \pmod{c}$ ------ (4)

By (4), $\xi^2 a^{(4p+2)} \equiv \eta^2 b^{(4p+2)} \pmod{c}$ --- (5)

~~.........~~ Putt (1) in (5),

$-\eta^2 b^{(p+2)} a^{3p} \equiv \eta^2 b^{(p+2)} b^{3p} \pmod{c}$.

$a^{3p} + b^{3p} \equiv 0 \pmod{c}$.

$(a^p+b^p)(a^{2p}-a^p b^p+b^{2p}) \equiv 0 \pmod{c}$.

Known!

(2) gives $a\xi \equiv -b\eta \pmod{c}$. ——— (6).

Putt (6) in (4), $0 \equiv -b\eta a^{2p} + b\eta b^{2p} \pmod{c}$,

So $a^{2p} - b^{2p} \equiv 0 \pmod{c}$

$(a^p+b^p)(a^p-b^p) \equiv 0 \pmod{c}$. Known!

(5) & (6) give $0 \equiv a^{4p} - b^{4p} \pmod{c}$

$0 \equiv (a^p+b^p)(a^{3p} - a^{2p}b^p + a^p b^{2p} - b^{3p})$.