①

FLT 1+2.

Let $p$ be an odd prime and let $x, y, z$ be relatively prime integers such that $x^p+y^p+z^p=0$. Let $p$ ~~not~~ divide $y$ or $x$.

$$\eta^p = \frac{x^p+z^p}{x+z} = x^{(p-1)} - z x^{(p-2)} + z^2 x^{(p-3)} - \dots + z^{(p-1)} \quad \text{---(1)}$$

Put $x = (x+y) - y$ in (1):

$$\eta^p \equiv y^{(p-1)} + z y^{(p-2)} + z^2 y^{(p-3)} + \dots + z^{(p-1)} \pmod{(x+y)} \quad \text{---(2)}$$

Similarly

$$\zeta^p \equiv x^{(p-1)} + z x^{(p-2)} + z^2 x^{(p-3)} + \dots + z^{(p-1)} \ (\text{''}) \quad \text{---(3)}$$

(2)+(3):

$$\zeta^p + \eta^p \equiv (x^{(p-1)} + y^{(p-1)}) + z^2(x^{(p-3)} + y^{(p-3)}) + \dots + 2z^{(p-1)} \ (\text{''}).$$

$$\equiv \left(\frac{x^{(p+1)} - z^{(p+1)}}{(x^2-z^2)}\right) + \left(\frac{y^{(p+1)} - z^{(p+1)}}{y^2 - z^2}\right) \pmod{(x+y)}.$$

$$\equiv \frac{x^{(p+1)}}{(x^2-z^2)} + \frac{y^{(p+1)}}{(y^2-z^2)} \pmod{(x+y)}.$$

$$(x^2-z^2)(y^p+z^p)(y-z) + (y^2-z^2)(x^p+z^p)(x-z)$$
$$\equiv x^{(p+1)}(y^2-z^2) + y^{(p+1)}(x^2-z^2) \pmod{(x+y)}.$$

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So $(x^2-z^2)y^p(y-z)+(y^2-z^2)(x^p)(x-z)$

$$\equiv (x^{(p+1)})(y^2-z^2)+y^{(p+1)}(x^2-z^2)\ (w).$$

Cancel:

$$\left(-zy^p(x^2-z^2)\right)+\left(-zx^p(y^2-z^2)\right)\equiv 0\ (\mathrm{mod}\,(x+y)).$$

$$z^3(x^p+y^p)-zx^2y^2(x^{(p-2)}+y^{(p-2)})\equiv 0\ (\mathrm{mod}\,(x+y)).$$