

late Feb, 2002

FLT 1+2, Use A for  $\xi$ .

Let  $p$  be an odd prime and let  $x, y, z$  be relatively prime integers such that  $x^p + y^p + z^p = 0$ . If  $p$  divides  $xyz$ , let  $p$  divide  $z$ . Write

$$\xi^p = \frac{y^p + z^p}{y+z}, a = (y+z), x = -a\xi, y = a - z.$$

Let  $|a| > 1$  because can't have  $|a| = |b| = 1$ .

So  $-a^p \xi^p + (a - z)^p + z^p = 0$ ,  $\xi + a$  are rel prime

So  $-a^p \xi^p + p a^p z^{p-1} \equiv 0 \pmod{a^{2p}}$

$$\xi^p \equiv p z^{p-1} \pmod{a^{2p}} \dots (1)$$

Similarly,  $\xi^p \equiv p y^{p-1} \pmod{a^{2p}} \dots (2)$

Actually,  $2\xi^p \equiv p y^{p-1} + p z^{p-1} \pmod{a^{2p}} \dots (2)$

~~(3)  $x(y+z) = a$~~

~~$$2(y^p + z^p) \equiv p(y^p + z^p + zy^{p-1} + yz^{p-1}) \pmod{a^{2p}}$$~~

~~$$0 \equiv (p-2)(y^p + z^p) + p y z (y^{p-2} + z^{p-2}) \pmod{a^{2p}} \dots (4)$$~~

~~$$0 \equiv (p-2) \left[ p(y+z)z^{p-1} - \frac{p(p-1)}{2}(y+z)^2 z^{p-2} \right]$$~~

~~$$+ p y z \left[ (p-2)(y+z)z^{p-3} - \frac{(p-2)(p-3)}{2}(y+z)^2 z^{p-4} \right] \dots (5)$$~~

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(1) xz - (2) xy =

$$z^p(z-y) \equiv p(z^{p-1}y - y^p) \pmod{a^p} \dots (6)$$

~~$$\div (z-y): z^p \equiv p(z^{p-1} + y^{p-1}) \pmod{a^p} \dots (5)$$~~

~~$$+ p y z(z^{p-3} + y z^{p-4} + \dots + y^{p-3}) \pmod{a^p}$$~~

~~because a is relatively prime to (z-y), as it divides (z+y), seeing~~

~~(5) x2 - (3):~~

~~$$0 \equiv p(z^{p-1} + y^{p-1}) + 2 p y z(z^{p-3} + \dots + y^{p-3}) \pmod{a^p}$$~~

(3) x(z-y) - (2) x2 =

$$0 \equiv -2p(z^p - y^p) + p(z-y)(z^{p-1} + y^{p-1}) \pmod{a^p}$$

cancel, transpose and divide by p, relatively prime to a,

$$(z^p - y^p) \equiv yz(z^{p-2} - y^{p-2}) \pmod{a^p}$$

$$\div (z-y):$$

$$z^{p-1} + yz^{p-2} + y^2z^{p-3} + \dots + y^{p-1}$$

$$\equiv yz^{p-2} + y^2z^{p-3} + \dots + y^{p-2} z \pmod{a^p}$$

Cancel:  $z^{p-1} + y^{p-1} \equiv 0 \pmod{a^p}$ ,  $2z^p \equiv 0 \pmod{a^p}$   
a rel prime to z, hence contradiction:  $2z^p \equiv 0 \pmod{a^p}$