(1).

FLT; H.

Let p be an odd prime and let x, y, z be relatively prime integers such that $p \nmid xy$ and such that $x^p + y^p + z^p = 0$.

Then $x^{(p-1)} \equiv y^{(p-1)} \equiv 1 \pmod{p^i}$, $i \geq 2$ and a maximal integer.

$$3y^{(p-1)} - 2x^{(p-1)} \equiv 2x^{(p-1)} - y^{(p-1)}$$

$$(3y^{(p-1)} - 2x^{(p-1)})^{(p-1)} \equiv (2x^{(p-1)} - y^{(p-1)})^{(p-1)} \quad \text{------} (1)$$

$$\{3^p y^{p(p-1)} - 2^p x^{p(p-1)}\}\{2x^{(p-1)} - y^{(p-1)}\}$$

$$\equiv \{2^p x^{p(p-1)} - y^{p(p-1)}\}\{3y^{(p-1)} - 2x^{(p-1)}\} \pmod{p}$$

$$-1 \times 3^p y^{(pp-1)} - 2^{(p+1)} x^{(pp-1)} + 2 \times 3^p x^{(p-1)} y^{p(p-1)} + 1 \times 2^p x^{p(p-1)} y^{(p-1)}$$

$$\equiv -2^{(p+1)} x^{(pp-1)} - 3 \times 1\, y^{(pp-1)} + 3 \times 2^p x^{p(p-1)} y^{(p-1)}$$

$$0 \equiv (3^p - 3) y^{(pp-1)} + 2 \times 2^p x^{p(p-1)} y^{(p-1)} - 2(3^p - 1) x^{(p-1)} y^{p(p-1)} + 2 \times 1\, y^{(p-1)} x^{(p-1)} \pmod{p}$$

$\times y$: $$0 \equiv (3^p - 3) y^{pp} + 2^{(p+1)} x^{p(p-1)} y^p - 2(3^p - 1) x^{(p-1)} y^{(pp-p+1)}$$

$$(3^p - 3)(x^p + z^p)^p + 2^{(p+1)} x^{p(p-1)} (x^p + z^p) \equiv -2(3^p - 1) x^{(p-1)} y^{(pp-p+1)} \pmod{p}$$

$$(3^p - 3 + 2^{(p+1)}) x^{pp} + (3^p - 3) z^{pp} + 2^{(p+1)} x^{p(p-1)} z^p$$

$$\equiv -2(3^p - 1) x^{(p-1)} y^{(pp-p+1)}$$

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$$(3^{p}-3+2^{(p+1)})(x+z) \equiv -2(3^{p}-1)y$$

$$4(x+y+z) \equiv 0 \pmod{p}.$$

Expand (1). $3^{(p-1)}-1 \equiv 0 \pmod{p}$. $\binom{p-1}{j} \equiv (-1)^{j} \pmod{p}$.

So $\displaystyle\sum_{j\neq 0,(p-1)} 3^{j} 2^{(p-1-j)} y^{j(p-1)} x^{(p-1-j)(p-1)}$

$$\equiv \sum_{j} 1^{j} 2^{(p-1-j)} y^{j(p-1)} x^{(p-1-j)(p-1)} \pmod{p}$$

(2) $\div 2y^{(p-1)}x^{(p-1)}$.

$$3\sum_{k=0,1,\dots,(p-3)} 3^{k} 2^{(p-3-k)} y^{k(p-1)} x^{(p-3-k)(p-1)} \equiv \sum_{k} 2^{(p-3-k)} y^{k(p-1)} x^{(p-3-k)(p-1)} \quad \text{---- (2)}$$

$$3\left(\{3y^{(p-1)}\}^{(p-2)} - \{2x^{(p-1)}\}^{(p-2)}\right)\left(y^{(p-1)}-2x^{(p-1)}\right)$$

$$\equiv \left(\{y^{(p-1)}\}^{(p-2)} - \{2x^{(p-1)}\}^{(p-2)}\right)\left(3y^{(p-1)}-2x^{(p-1)}\right)$$

$$3(3^{(p-2)}-2^{(p-2)})(1-2) \equiv (1-2^{(p-2)})(3-2) \pmod{p}.$$

$$-(3^{(p-1)}-2^{(p-1)}-2^{(p-2)}) \equiv (1-2^{(p-2)})$$

$$2^{(p-2)} \equiv 1-2^{(p-2)} \pmod{p}.$$

$$2^{(p-1)} \equiv 1 \pmod{p}.$$