On FLT and a result of Falting.

Let $n \geq 3$ be integral and let x, y, z, x', y', z' be positive integers such that

$$x^n + y^n = z^n$$

and $x'^n + y'^n = z'^n$.

Then $(zz')^n = (xx')^n + (xy')^n + (yx')^n + (yy')^n$

Also $(z^n - y^n)(z'^n - y'^n) = (xx')^n$,

ie $(zz')^n = (xx')^n - (yy')^n + (yz')^n + (zy')^n$ --- (1)

Similarly,

$$(zz')^n = (xx')^n + (yy')^n + (xz')^n + (zx')^n$$

So $-2(xx')^n + (xz')^n + (zx')^n$ --- (2)

$= -2(yy')^n + (yz')^n + (zy')^n$

This is, $x^n y'^n + y^n x'^n$

$= y^n x'^n + x^n y'^n$.

I had hoped to prove Fermat's equation for exponent n has at most one solution in positive integers (a result due to Falting and Granville).

equivalent to ②

(1) is equivalent to $(zz')^n = (xx')^n + (yy')^n + (yx')^n + (xy')^n$ --- (3)

We had this at the beginning.

$(1)+(2) = 2\times(3)$:

$(yz')^n + (zy')^n + (xz')^n + (zx')^n$

$= 2\{(xx')^n + (yy')^n + (yx')^n + (xy')^n\}$.

~~Let~~ $p = n$, p an odd prime.

Then

$(yz' + zy' + xz' + zx') \equiv 2(xx' + yy' + yx' + xy') \pmod{p}$

~~However, if $p^k \mid z$, $p^{(k+1)} \nmid z$, $k \geq 1$ and integral~~
~~and similarly k'), then x, y, x', y' need not~~
~~all be positive but $k = k'$. A start to~~
~~proving there is at most one solution.~~

$(1)-(2)$: $0 = 2\{(xx')^n - (yy')^n\} + (z')^n(y^n - x^n) + z^n(y'^n - x'^n)$

~~So $(xx')^n \equiv (yy')^n \pmod{p^{nk}}$~~
~~$\equiv [(x+y) - x]^n [(x'+y') - y']^n$~~
~~$\equiv [c^p p^{p(k-1)} - x]^p [c'^p p^{p(k-1)} - y']^p$~~

So $0 \equiv 2\{(xx')^p - (yy')^p\} - 2\{(z'x)^p + (zx')^p\} \pmod{p^{2pk}}$

Good!

On FLT and a result of Falting.

- So $(xx')^p - (xz')^p \equiv (yy')^p + (zx')^p$

$$-(xy')^p \equiv (yy')^p + (zx')^p$$

$$-(zx')^p \equiv (xy')^p + (yy')^p \equiv (zy')^p.$$

$$(x'^p + y'^p)\, z^p \equiv 0 \pmod{p^{2pk}}$$

~~(1)+(3)~~ $n = 2p$: y, y' even, x, x', z, z' odd.

$2^\lambda \mid y, y'$, $\lambda \geq 1$ and integral.

- (1)+(2) give

$$2(xx')^{2p} \equiv (xz')^{2p} + (zx')^{2p} \pmod{2^{2p\lambda}} \quad \text{---(4)}$$

This reduces to an identity! useless!

(1)+(2):

$$2(zz')^{2p} \equiv (xz')^{2p} + (zx')^{2p} \pmod{2^{2p\lambda}} \quad \text{---(5)}$$

$$\equiv 2(xx')^{2p} \pmod{2^{2p\lambda}}$$

$$(xx')^{2p} + (yx')^{2p} + (xy')^{2p} + (yy')^{2p} \equiv (xx')^{2p} \pmod{2^{2p\lambda-1}} \quad \text{---(6)}$$

But (7) is valid mod $2^{2p\lambda}$ ---(7)

Follows also from (3), not just mod $2^{2p\lambda-1}$

- What are we to make of this?

~~(1) gives $(zz')^n = (xx')^n + (yy')^n$~~

Now $(x^n - x'^n)(y^n - y'^n)$

$$= (xy)^n - (x'y)^n - (xy')^n + (x'y')^n$$

$$= (xy)^n + (x'y')^n - (zz')^n + (xx')^n + (yy')^n$$

$$= (xy)^n + (x'y')^n + (xx')^n + \cancel{(yy')^n} \quad \text{---- (8)}$$

$$+ (xx')^n - \cancel{(yy')^n} - (xz')^n - (zx')^n \quad \text{--- (9)}$$

(8) gives $(zz')^{2p} \equiv (xx')^{2p} \pmod{2^{2p\lambda}}$, already known!

(9) gives $2(xx')^p \equiv (xz')^{2p} + (zx')^{2p} \pmod{2^{2p\lambda}}$ — already known!

(9) is $-(x^n - x'^n)^2 - (x^n - x'^n)(z'^n - z^n)$

$$= (xy)^n + (x'y')^n + 2(xx')^n - (xz')^n - (zx')^n$$

$$= x^n(z^n - x^n) + x'^n(z'^n - x'^n) + 2(xx')^n - (xz')^n - (zx')^n$$

Put (8) in terms of y, y', z, z', eliminating x, x'. Any use?