1a

Legendre's Thm for FLT2.

Let p be an odd prime and let x, y, z be relatively prime integers such that $p | z^k$, $p^{(k+1)} \nmid z$, $k \geq 1$ and integral and such that $x^p + y^p + z^p = 0$.

Show $y+z \neq 1$. Suppose $(y+z) = 1$. Then

$y = 1 + cp^k\zeta$; $z = -cp^k\zeta$,

$(x+y) = c^p p^{(pk-1)}$. So $x = c^p p^{(pk-1)} - (1+cp^k\zeta)$.

So $0 = x^p + y^p + z^p$

$= [c^p p^{(pk-1)} - (1+cp^k\zeta)]^p + (1+cp^k\zeta)^p - c^p p^{pk}\zeta^p$

$= (c^p p^{(pk-1)})^p - p(c^p p^{(pk-1)})^{(p-1)}(1+cp^k\zeta) + \cdots$

$\cdots + p(c^p p^{(pk-1)})(1+cp^k\zeta)^{(p-1)} - c^p p^{pk}\zeta^p$

Modulo $p^{(pk+2k)}$, we get

$c^p p^{pk}(-\zeta^p + 1 + cp^k\zeta(p-1)) \equiv 0$ ------- (1)

~~... contradiction ... result.~~

So $\zeta^p \equiv 1 \pmod{p^k}$. ~~Not~~ good enough.

Try $x+z = b^p$, so $x = b^p + cp^k\zeta$, $z = -cp^k\zeta$

$x = c^p p^{(pk-1)} - (b^p + cp^k\zeta)$. Then, analogous to

??

(1), we get

$$c^l p^{pk}\left(-\zeta^p + (b^p)^{(p-1)} + (p-1)cp^k\zeta(b^p)^{(p-2)}\right) \equiv 0. \quad \text{--- (2)}$$

(2) − (1):

$$(b^{p(p-1)} - 1) + (p-1)cp^k\zeta(b^{p(p-2)} - 1) \equiv 0 \pmod{p^{2k}} \quad \text{--- (3)}$$

So $b^{p(p-1)} \equiv 1 \pmod{p^k}$, i.e. $b^{(p-1)} \equiv 1 \pmod{p^{k-1}}$. --- (4)

(3) × b:

$$(b^{(p^2-p+1)} - b) + (p-1)cp^k\zeta(b^{(p-1)(p-1)} - b) \equiv 0 \ (u).$$

Use (4):

$$b(b^{p(p-1)} - 1) \equiv (p-1)cp^k\zeta(b-1) \pmod{p^{2k-1}} \quad \text{--- (5)}$$

~~÷ (b−1): check $b \not\equiv 1 \pmod p$.~~

~~$\equiv b^{p(p-1)} + b^{(p-1)(p-1)+(p-2)} + b^{(p-1)(p-1)+(p-3)} + \dots + b \equiv (p-1)cp^k\zeta \ (u).$~~

~~$\equiv p + p(b^{(p-2)} + b^{(p-3)} + \dots + b) \pmod{p^{k-1}} \equiv 0$~~

~~$1 + \dfrac{b(b^{(p-2)} - 1)}{(b-1)} \equiv 0$ This is $b^{(p-1)} \equiv 1 \pmod{p^{k-1}}$ known!~~

(1) × $b^{p(p-2)}$ − (2):

$$0 \equiv c^l p^{pk}\left((-\zeta^p + 1)b^{p(p-2)} + \zeta^p b^{p(p-1)}\right)$$

??

$$\zeta^p(b^{p(p-2)} - 1) \equiv b^{p(p-2)}(1 - b^p) \pmod{p^{2k}} \quad \text{--- (6)}$$