ON THE NON-EXISTENCE OF NON-EUCLIDEAN GEOMETRIES AND THE NON-INDEPENDENCE OF EUCLID'S PARALLEL POSTULATE

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Euclid wrote the thirteen books of *Elements* over 2,000 years ago and it remained the standard reference on geometry until the discovery of non-Euclidean geometries by Riemann and by Lobachevsky and by Bolyai early in the nineteenth century. The present author will prove that the belief based on a mis-understanding.

Euclid's fifth postulate, the parallel postulate, states:

If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the angles are the two right angles, less than two right angles.

\[
\text{Angle } BAC + \text{Angle } DBA < (\text{two right angles})
\]

then \( AC \) meets \( BD \).
Many attempts have been made to prove Euclid's parallel postulate, all without success. The task is to prove the parallel postulate from the other assumptions of Euclid. Finally, Toulon [ ] proved that it could not be done: that was published in 1870 and will be repeated here for the convenience of readers. However, the present author will give a proof of Euclid's parallel postulate using only Euclid's other assumptions. The existence of such a proof implies that Euclidean geometry is inconsistent, meaning it a result and its negation can both be proved.