1. A partial differential equation with no solution, no matter what the boundary conditions are, by John H. UrSELL usually of Kingston, Ontario, Canada research. This article shows that the partial differential equation
\[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0 \]
has no solution.

A solution in formal power series is assumed to exist and we obtain a contradiction to its being convergent. Let, m n be a term in that convergent formal power series for u in a certain neighborhood of the xy-plane. Then that...
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term $A \frac{\partial u}{\partial x} \frac{\partial u}{\partial y}$ implies that $\frac{\partial u}{\partial y} \frac{\partial u}{\partial x}$ has a term $A^2 (m)(n) e^{\lambda m-1}$, which must also be in

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$$

This implies that $\frac{\partial u}{\partial x}$ has a term $KA^2 (m)(n) e^{\lambda m-1}$, where $K$ and $A$ are constants. Also, $\frac{\partial u}{\partial y}$ has a term $(1-K) A^2 (m)(n) e^{\lambda m-1}$.

If $\frac{\partial u}{\partial x}$ has that term mentioned, then $u$ has a term $KA^2 (m)(n) e^{\lambda m-1} y^{\lambda m-1}$, With
A partial differential equation with no solution, no matter what the boundary conditions are, this term \( \frac{K A^2 (m^n)}{(am)(am+1)} \) \( \frac{y}{y} \) in \( \Omega \) then \( \frac{\partial u}{\partial x} \) has a term\[
\frac{K A^4 (m^n)}{(am)^2(am+1)^2} \frac{y}{y} \frac{(am+1)(am+1)}{(am)(am+1)} \]
This in turn implies that \( \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y \partial x} \) has such a term. Thus \( \frac{\partial u}{\partial x} \) has a term\[
\frac{L K A^4 (m^n)}{(am)^2(am+1)^2} \frac{(am+1)(am+1)}{(am)(am+1)} \frac{y}{y} \frac{(am+1)(am+1)}{(am)(am+1)} \]
and \( \frac{\partial u}{\partial y} \) has a term.
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\[(1 - L) K^2 A^4 (m^2) (n^2) (\lambda m + 1) (\lambda n) \frac{\partial^2 u}{\partial m^2} \frac{\partial^2 u}{\partial n^2} = \varepsilon (\lambda m + 1) y^{(4\eta)}(4m)\]

Thus, it has a term:

\[(\lambda m + 1) y^{(4\eta)}(4m)\]

Now consider the convergent formal power series for \(u\). Presumably, it is convergent in a rectangle in the \(xy\)-plane. With any \(\eta\)-value in this rectangle, the power series in \(\varepsilon\) only is convergent for all \((x, y)\) in the rectangle. Similarly, for particular values of \(x_0\) and the resulting
A partial differential equation with no solution no matter what the boundary conditions are power series in y only but with coefficients involving \( x \). It is believable that some points in the \((x,y)\)-plane but outside that rectangle also bring convergence of that power series in \( x \) and \( y \).

Now let \((a, b)\) be in that rectangle mentioned as being some of the points where the power series converges.

Let that rectangle be \( A_1 \leq x \leq A_2 \) plus \( B_1 \leq y \leq B_2 \).

The construction of the rectangle indicates that \( A_1 > 0 \leq A_2 \) and \( B_1 < 0 \leq B_2 \).
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Now write \( X = (x - A_2 + A_1) \) and \( Y = (y - B_2 + B_1) \). Then the point \( X = 0, Y = 0 \) is not inside the rectangle in the \((x, y)\)-plane but \( x = 0, y = 0 \) is also permits this. This means that \( x = 0, y = 0 \) being in the rectangle implies that the point \( X = 0, Y = 0 \) is also in the rectangle. But we have seen that the point \( X = 0, Y = 0 \) is not in the rectangle. The contradiction shows that the rectangle is the whole \((x, y)\)-plane.
A partial differential equation with no solution no matter what the boundary conditions are. No solution exists if there is a convergent power series on the whole or the \((x,y)\)-plane then for any value \(y = b\), the power series is absolutely convergent for all \(x\).