Dec 7

On several weaknesses of quantum mechanics.

Let \( A \) be a linear operator in Hilbert space, eigenvalues \( \lambda \), eigenvectors \( \mathbf{x} \), \( \| \mathbf{x} \| < \infty \), \( \lambda \neq 0 \).

Then \( A \mathbf{x} = \lambda \mathbf{x}. \)

So \( \mathbf{I} \mathbf{x} = \lambda A^{-1} \mathbf{x} \),

\[ A^{-1} \mathbf{x} = \frac{1}{\lambda} \mathbf{x}. \]

Let \( A = [a_{ij}] \)

\[ a_{ij} = 0, \quad i \neq j \]

\[ a_{ii} = \frac{1}{\sqrt{i}}, \quad i = 1, 2, \ldots \]

Then \( A^{-1} = [b_{ij}] \),

\[ b_{ij} = 0, \quad i \neq j \]

\[ b_{ii} = \sqrt{i}, \quad i = 1, 2, \ldots \]

Let \( \mathbf{x} = (c_i) \), \( c_i = i^{-2/3} \), \( i = 1, 2, \ldots \), \( \| \mathbf{x} \| \infty < \infty \).

But \( \| A^{-1} \mathbf{x} \| = \sqrt{\sum_i |c_i^{1/3}|^2} = \infty. \)

\( A^{-1} \) is unbounded. Inversion not continuous for non-singular linear operators in Hilbert space form a group but not a topological group.
Arens proved that a topological semigroup on a finite-dimensional Euclidean space which is a topological group is invalid for Hilbert space.

So in quantum mechanics, we need topological semigroups which are groups, but not topological groups. Maybe this change will bring new important results to end the drought lamented by Lee Smolin, "The Trouble with Physics."