THE TRADITIONAL AMALGAM OF QUANTUM THEORY AND SPECIAL RELATIVITY ADMITS A MAXIMUM PHOTON FREQUENCY

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We show that in certain circumstances the traditional amalgam of quantum theory and special relativity admits a maximum photon frequency. This in turn shows that the said amalgam is self-contradictory.

Let a photon with frequency $\nu$ move along the direction of the unit vector $\hat{u}$ and hit an electron and rebound in the direction $\hat{v}$ backwards with frequency $\nu'$.

Let the electron be moving before the collision in the direction of the unit vector $\hat{f}$ perpendicular to $\hat{u}$ with

\[ \hat{f} \times \hat{u} = \hat{v} \]
velocity  \( v \) cm/sec, \( v > 0 \). After the collision, let the electron move on with velocity \( w \) cm/sec, \( w > 0 \), in the direction of the unit vector \( \hat{t} = \hat{c} \sin(\phi) + \hat{f} \cos(\phi) \), inclined at angle \( \phi \) to its previous path. Let \( \hat{n} = -\hat{c} \cos(\phi) + \hat{f} \sin(\phi) \) be the unit vector perpendicular to \( \hat{t} \). Let the mass of the electron be \( m \) gms, the velocity of light be \( c \) cm/sec.

Conservation of momentum yields:

\[
\frac{hv}{c} \hat{t} + \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} \hat{f} = -\frac{hv}{c} \hat{t} + \frac{mw}{\sqrt{1 - \frac{w^2}{c^2}}} \hat{n}
\]

This yields:

\[
\frac{hv}{c} = \frac{mw}{\sqrt{1 - \frac{w^2}{c^2}}} \sin(\phi)
\]  \hspace{1cm} (1)

and

\[
\frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{mw}{\sqrt{1 - \frac{w^2}{c^2}}} \cos(\phi)
\]  \hspace{1cm} (2)

Eliminate \( w \) from (2) and (3):
The collision is elastic and thus energy is conserved. Thus yields:

\[
\frac{h(y+y') \cos(\phi)}{c} = \frac{mv}{\sqrt{1-v^2/c^2}} \sin(\phi) - \tag{4}
\]

\[
= \frac{h \nu' c + mc^2}{\sqrt{1-v^2/c^2}} + mc^2 n - \tag{5}
\]

This gives:

\[
\frac{h(y-y')} + mc^2(1+\cos(\phi)) = \frac{mc^2}{\sqrt{1-v^2/c^2}} \sin(\phi) - \tag{6}
\]

and

\[
\frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 \sin(\phi) = \frac{mc^2}{\sqrt{1-v^2/c^2}} \cos(\phi) - \tag{7}
\]

Eliminate \( n \) from (6) and (7),
\[ r(\nu - \nu') + mc^2 \left( 1 + \cos(\phi) \right) \left[ \cos(\phi) \right] \]
\[ = \left[ \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2 \sin(\phi) \right] \sin(\phi) \]

Rewrite (8) as:
\[ \left\{ r(\nu - \nu') \right\} \cos(\phi) + mc^2 \left( \cos^2(\phi) + \sin^2(\phi) \right) \]
\[ = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \sin(\phi) \]
This is:
\[ \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \sin(\phi) \left\{ r(\nu - \nu') \right\} + mc^2 \left( \cos^2(\phi) + \sin^2(\phi) \right) \]
\[ = mc^2 \quad \text{(9)} \]

Recall that (4) is:
\[ \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} \sin(\phi) = \frac{r(\nu + \nu')}{c} \cos(\phi) \]

Put (4) in (9) to eliminate \( \sin(\phi) \):
\[ \left( \frac{c^2}{\nu} \right) \left( \frac{r(\nu + \nu')}{c} \right) \cos(\phi) - \left\{ r(\nu - \nu') + mc^2 \right\} \cos^2(\phi) \]
\[ = mc^2 \quad \text{(10)} \]
This is:
\[ \cos(\phi) \left[ h \left( v + v' \right) - h \left( v - v' \right) v - mc^2 v \right] = mc^2 v \quad \text{(11)} \]

(11) gives
\[ 2\cos^2 \left( \frac{\phi}{2} \right) = 1 + \cos(\phi) \]
\[ = \frac{\left[ h(v+v') c - h(v-v') v \right]}{\left[ h(v+v') c - h(v-v') v - mc^2 v \right]} \quad \text{(12)} \]

Suppose that the frame of reference \( \varepsilon \) cm/sec is such that \( \frac{\varepsilon}{c} \approx 0 \). Then
\[ 2\cos^2 \left( \frac{\phi}{2} \right) \approx \frac{2hv'c + hv \varepsilon}{h(v+v')c - mc^3 + hv \varepsilon} \]

Thus
\[ \sec^2 \left( \frac{\phi}{2} \right) \approx (2 - \frac{mc^2}{hv}) \quad \text{(13)} \]

but in (4),
\[ \tan(\phi) = \frac{h(v+v')}{mvc} \sqrt{1 - \frac{v^2}{c^2}} \quad \text{(14)} \]
(14) yields when \( v \cong c \):
\[
\tan(\theta) \approx 0.
\]
Thus \( \phi \approx 0 \) \hfill (15)

Put (15) in (13) and obtain:
\[
' = \sec^2(\theta) = \left( 2 - \frac{mc^2}{\hbar v'} \right) \hfill (16)
\]
This yields
\[
v' = \frac{mc^2}{\hbar} \hfill (17)
\]

The corresponding wavelength
\( \lambda' \) is
\[
\lambda' = \frac{c}{v'} = \frac{\hbar}{mc}. \hfill \text{This is the Compton wavelength for the electron.}