Relativity.

Let space and time be discrete, minimum step in length being \( \Delta x \) cm and minimum step in time being \( \Delta t \) sec. Let the light travel \( c \Delta t \) cm in time \( \Delta t \) sec. Let a particle of mass \( m \) gms move in a straight line with velocity \( v \) cm/sec where

\[
C \Delta t = v \Delta t + \Delta x \quad \text{(1)}
\]

A photon comes in perpendicular to the path of the particle. The photon has frequency \( \nu \) and wavelength \( \lambda = \frac{c}{\nu} \).

\[
C = \lambda \nu = \frac{c}{\nu} \nu \quad \text{(2)}
\]

The photon impacts the particle, then retraces its path but with frequency \( \nu' \) and wavelength \( \lambda' \).

\[
C = \nu' \lambda' \quad \text{(3)}
\]

The particle moves off with maximum velocity \( V \) in a direction at angle \( \theta \) to the original direction of the photon's motion. The collision is elastic.

Equations of motion in the direction of the photon's original path:

\[
\frac{\hbar \nu}{C} = -\frac{\hbar \nu'}{C} + \frac{mv \cos \theta}{c} \quad \text{(4)}
\]

\[
\hbar \nu' = \hbar \nu + \frac{mc^2}{\sqrt{1 - \frac{v^2 \cos^2 \theta}{c^2}}} \quad \text{(5)}
\]
Equations of motion in the direction of the particle’s original path:

$$\frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{m v \sin \theta}{\sqrt{1 - \frac{v^2 \sin^2 \theta}{c^2}}}$$

$$(6)$$

$$\frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{mc^2}{\sqrt{1 - \frac{v^2 \sin^2 \theta}{c^2}}}$$

$$(7)$$

So $\sin \theta = 1$, $\theta = \frac{\pi}{2}$

Put (8) in (4) and (5):

$$\frac{\hbar \gamma}{c} = - \frac{\hbar \gamma}{c}$$

and $\hbar \gamma = \hbar \gamma + mc^2$

$$(9)$$

So, by (9) + (10),

$$\Delta \gamma = mc^2$$

$$(10)$$

$$\nu = \frac{mc^2}{\Delta \gamma}$$

$$(11)$$

(11) $\times \Delta x$:

$$c = \nu \Delta x = \frac{mc^2}{\Delta \gamma} \Delta x$$

So $m \gamma \Delta x = \frac{mc^2}{\Delta \gamma}$

$$(12)$$

Is $m$ the minimum unit of mass in relativistic quantum mechanics? Then $\Delta \gamma$ as the minimum unit of energy $\nu = \frac{mc^2}{\Delta \gamma}$ is maximum frequency?
\[ \Delta x = \chi = \frac{c}{2} = \frac{mc^2}{2\hbar} = \frac{2\hbar}{mc} \] is the minimum length. What is \( m \)? \( m \) is presumably the mass of the largest particle, presumably the nucleus of a transuranium element.

\[ \Delta p = (\text{minimum unit of momentum}) = m^* \Delta v \]
\[ \Delta x \Delta p \geq \hbar \quad \text{m}^* (\text{minimum unit of mass}) \]
\[ \Delta x \left( \frac{m^* \Delta x}{c} \right) \geq \hbar \]
\[ m^* \geq \frac{\hbar c}{(\Delta x)^2} \]
\[ m^* \geq \frac{m^2 \hbar c^3}{4\hbar^2} = \frac{m^2 c^3}{4\hbar} \]

Set \( m^* = \frac{m^2 c^3}{4\hbar} \)

But \( m^* < m \)
\[ m^2 10^{57} < m \]

\[ m < 10^{-57} \text{ g} \quad \text{Minimum unit of mass} \]
\[ \Delta E \Delta t \geq \hbar \quad \text{Energy of rest mass of largest particle} \]

Minimum unit of energy \( \Delta E \) is less than \( 9 \times 10^{-37} \text{ ergs} \)
\[ \Delta t \geq \frac{k}{\Delta E} \geq \frac{\hbar}{9 \times 10^{-37}} = \frac{2 \times 10^{-26}}{9 \times 10^{-37}} = \frac{2}{9 \times 10^{11}} \text{ seconds} \approx 2 \frac{1}{2} \text{ years} \]

What can this be? Is it the interval until the universe splits, or until the path of history separates? There is a particle (electron or proton or neutron) with mass over \(10^{-57} \text{ g m}^2\). So what can the inequality (18) mean?

Take \( M \) to mean minimum mass \( m^* \), \( m = m^* \).

\[ \nu = \frac{mc^2}{\hbar} = \frac{10^{-57} \times 9 \times 10^{20}}{2 \times 6.7 \times 10^{-27}} \]

\[ = \frac{2.7 \times 10^{-10}}{4} = 6.7 \times 10^{-11} \text{ cycles/sec} \]

\[ \text{Max} \lambda = \frac{c}{\nu} = \frac{3 \times 10^{10}}{6.7 \times 10^{-11}} = \frac{9 \times 10^{21}}{20} = 4.5 \times 10^{20} \text{ cm} \approx \text{distance to Andromeda nebula, I think} \]

Go back to (11) and (12):

\[ \Delta x = \frac{2 \hbar}{mc} \text{ (cm)} \]

(maximum possible frequency of photon) \( \nu = \frac{mc^2}{\hbar} \) (cycles/sec)

There is a minimum length!!

Need to find \( \min (m) \) and \( \max (m) \)

Is the energy + the momentum equal different from above?
Relativity.

Let the photon give the particle a velocity $u$ transversely. The impulsive force from the photon is

$\frac{\gamma U + U' \gamma}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{mu}{\sqrt{1 - \frac{v^2}{c^2}}}$

Or $\gamma U + U' \gamma = mUc$

Energy transversely:

$\gamma U \gamma' = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$

So (10) + (11) added give:

$2\gamma U = mUc + \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$

Then $\frac{V}{\sin(\phi)} = u \cos(\phi)$

and $V = \left( \text{sum of } u \cos(\phi) \text{ and } \right)$

$$u \sin(\phi)$$

$$= \frac{V \cos(\phi)}{1 + \frac{U \cos(\phi) \sin(\phi)}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

By (13) $u = V \tan(\phi)$

(15) in (14):

$$V = \frac{V \cos(\phi) + V \tan(\phi) \sin(\phi)}{1 + \frac{V \cos(\phi) \sin(\phi)}{\sqrt{1 - \frac{v^2}{c^2}}}}$$
\[ 1 = \frac{\sec(\phi)}{1 + \frac{v^2}{c^2} \sin^2(\phi)} \]

\[ \frac{v^2}{c^2} \sin^2(\phi) = \sec(\phi) - 1 = \frac{1 - \cos(\phi)}{\cos(\phi)} \]

\[ \frac{v^2}{c^2} = \frac{1 - \cos(\phi)}{\cos(\phi)} \times \frac{1}{1 - \cos^2(\phi)} \]

\[ \frac{u^2}{c^2} = \frac{u^2}{v^2} \cdot \frac{v^2}{c^2} \]

\[ = \frac{\tan^2(\phi)}{\cos(\phi)(1 + \cos(\phi))} \]

\[ (16) + (17) \text{ in (15)}: \]

\[ \frac{u^2}{m} = uct + \frac{v^2}{c^2} \]

\[ = \frac{\tan(\phi)}{\cos(\phi)(1 + \cos(\phi))} \cdot \frac{1}{1 - \frac{v^2}{c^2}} \]

\[ c^2 + \sqrt{1 - \frac{v^2}{c^2}} \]
\[ \text{Relativity} \]

\[ H = \frac{c \tan \theta}{\sqrt{\cos \theta (1 + \cos \theta)}} + \frac{c^2 \sqrt{\alpha}}{\sqrt{1}} \]

\[ \approx \frac{c \phi}{\sqrt{\alpha}} + \frac{c^2 \sqrt{\alpha}}{\sqrt{1}} \]

\[ \approx \sqrt{\alpha} c^2 \]

\[ \frac{\alpha \nu}{m} \approx \sqrt{\alpha} c^2 \]

\[ \gamma = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \]

Let the particle be going at \( v \) cm/sec and move at \( \alpha \nu \) cm/sec after the impact with the photon. Then let it move at \( w \) cm/sec after impact.

Then \( \frac{\alpha \nu}{c\sqrt{1 - \frac{v^2}{c^2}}} + \frac{\alpha \nu}{c\sqrt{1 - \frac{w^2}{c^2}}} = \frac{mu}{\sqrt{1 - \frac{w^2}{c^2}}} \)

\[ \alpha \nu = \frac{mc^2}{\sqrt{1 - \frac{w^2}{c^2}}} \]
\[(22) + \frac{1}{c \sqrt{1 - \frac{v^2}{c^2}}} \times (23):\]

\[\frac{kv}{c \sqrt{1 - \frac{v^2}{c^2}}} + \frac{hv}{c \sqrt{1 - \frac{w^2}{c^2}}},\]

\[= \frac{mu}{\sqrt{1 - \frac{w^2}{c^2}}} + \frac{mc}{\sqrt{1 - \frac{w^2}{c^2}}}.\] ——— (24)

\[w \sin(\phi) = u \cos(\phi),\] ——— (25)

\[W = \frac{V \cos(\phi) + U \sin(\phi)}{1 + UW \sin(\phi) \cos(\phi)}\] ——— (26)

But (25) in (26) gives:

\[W = \frac{V \sec(\phi)}{1 + \frac{V^2}{c^2} \sin^2(\phi)}\] ——— (27)

\[W \leq V\] because \(v\) is maximum attainable velocity of a particle.

\[\frac{v^2}{c^2} < \frac{1}{\cos(\phi) \left(1 + \cos(\phi)\right)}\] ——— (28).
Relativity

(25) and (28) yield
\[
\frac{u^2}{c^2} = \frac{u^2}{c^2} \cdot \frac{c^2}{v^2} < \frac{\tan^2 \theta}{\cos \theta (1 + \cos \theta)}.
\]

Which means
\[
\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} > \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}
\]

(30) in (34) gives
\[
\frac{2h\nu}{mc^2} > \frac{mc (\nu/c)}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{mc}{\sqrt{1 - \frac{u^2}{c^2}}}
\]

\[
\frac{2h\nu}{mc^2} > \frac{u}{c} + \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

\[
\frac{2h\nu}{mc^2} > \frac{\tan \theta}{\sqrt{\cos \theta (1 + \cos \theta)}} + \frac{1}{\sqrt{1 - \cos \theta (1 + \cos \theta)}}
\]

Let \( v \) be least frequency (weakest radio).

\[
\frac{2h\nu}{mc^2} > \frac{\Phi}{\sqrt{\frac{v^2}{c^2} + \frac{v^2}{c^2}}} = \frac{\nu}{\sqrt{2}}
\]

\[
\frac{mc^2}{\nu^2}
\]

(31)
but $u \leq v$ because $v$ is the maximum velocity for a particle. So $0 < \tan(\phi) \leq 1,$ $0 < \phi \leq \pi/4.$

So $\cos(\phi) \approx 1,$ $\sin(\phi) \approx \phi,$ $\tan(\phi) \approx \phi.$

By (28), \[ \frac{v^2}{c^2} \leq \frac{1}{\cos(\phi)(1 + \cos(\phi))} \leq \frac{1}{(\frac{1}{\sqrt{2}})(1 + \frac{1}{\sqrt{2}})} = \frac{1}{\sqrt{3}} \]

\[ \frac{v^2}{c^2} \leq \frac{2}{(\sqrt{2} + 1)} = \frac{2(\sqrt{3} - 1)}{(2 - 1)} = 2(\sqrt{3} - 1) \]

$V < 0.9c$ or less.

$0 < V < 0.91c$ \hspace{1cm} (33)

All this is special relativity. Under acceleration in general relativity, it may be possible.

What about a frame of reference travelling at $99\%$ of $c,$ etc.

Use calculus:

\[ W = \frac{u + v}{1 + \frac{uv}{c^2}} \quad 0 < u, v < c \]

\[ \frac{dW}{du} = \frac{1}{(1 + \frac{uv}{c^2})} - \frac{(u + v) \left( \frac{v}{c^2} \right)}{(1 + \frac{uv}{c^2})^2} = 0 \]

So \[ \frac{1 + \frac{uv}{c^2}}{(1 + \frac{uv}{c^2})^2} = (u + v) \left( \frac{v}{c^2} \right) \]

\[ (1 + \frac{uv}{c^2}) = (u + v) \left( \frac{v}{c^2} \right) \]

\[ (1 + \frac{uv}{c^2}) = \left( \frac{v}{c} \right)^2 \] \hspace{1cm} (34)
Similarly, \( \frac{dw}{dv} = 0 \) gives

\[
(1 + \frac{uv}{c^2}) = (u + v)(\frac{v}{c^2})
\]

\((34) + (35)\) show \( u = v \) and

\[
(1 + \frac{u^2}{c^2}) = \frac{2u^2}{c^2}
\]

\( \Rightarrow u = c = v. \)

Not the same as \( q(\% \text{ of } c) \).

\[(27) \text{ is } W = \frac{v \sec(\theta)}{1 + \frac{v^2}{c^2} \sin^2(\theta)}.
\]

Minimise \( W \).

Maximise \( W \).

\( v \) constant, \( \phi \) varies \( \Rightarrow \) makes \( \phi \) vary.

\[0 = \frac{dw}{d\phi} = v \left\{ \sec(\phi) \tan(\phi) \left[ 1 + \frac{v^2}{c^2} \sin^2(\phi) \right] - \frac{1}{c^2} \frac{v^2}{c^2} \right\} \frac{v \cos^2(\phi)}{\sin(\phi)} (1 + \ldots)^2\]

\[\left(\frac{\cos^2(\phi)}{\sin(\phi)}\right) \times \frac{\cos^2(\phi)}{\sin(\phi)}.
\]

\[0 = \left\{ 1 + \frac{v^2}{c^2} \sin^2(\phi) \right\} - 2 \frac{v^2}{c^2} \cos^2(\phi).
\]

\[= \left\{ 1 + \frac{v^2}{c^2} \sin^2(\phi) \right\} - 2 \frac{v^2}{c^2} (1 - \sin^2(\phi)).\]
\[
\frac{3v^2}{c^2 \sin^2 \theta} = \left(\frac{2v^2}{c^2} - 1\right)
\]
\[
\sin^2 \theta = \left(\frac{2}{3} - \frac{c^2}{3v^2}\right)
\]

But \(\sin^2 \theta \geq 0\), so \(v^2 \geq \frac{1}{2}c^2\). --- (67)

There is only a max or min when \(v \geq \frac{\sqrt{2}}{2}c\) (except for \(\sin \theta = 0\), i.e. \(\theta = 0\)). Then

\[
1 + \frac{v^2}{c^2 \sin^2 \theta} = 1 + \frac{2}{3} \frac{v^2}{c^2} - \frac{1}{3} = \frac{2}{3} \left(1 + \frac{v^2}{c^2}\right)
\]

and \(\cos \theta = \sqrt{1 - \left(\frac{2}{3} - \frac{c^2}{3v^2}\right)^2}\), so

\[
\sec \theta = \frac{3v^2}{\sqrt{1 - \left(\frac{2}{3} - \frac{c^2}{3v^2}\right)^2}} = \frac{3v^2}{\sqrt{9v^4 - (2v^2 - c^2)^2}}
\]

\[
= \frac{3v^2}{\sqrt{(3v^2 - (2v^2 - c^2))(3v^2 + (2v^2 - c^2))}}
\]

\[
= \frac{3v^2}{\sqrt{v^2 + c^2)(5v^2 - c^2)}
\]

So

\[
\frac{W}{U}\max_{\text{or min}} = \frac{\sec \theta}{1 + \frac{v^2}{c^2} \sin^2 \theta} = \frac{3v^2}{\sqrt{9v^2c^2 - (2v^2 - c^2)(5v^2 - c^2)}}
\]
Relativity.

Let \( v \to c \). Then

\[
\frac{\gamma}{\sqrt{1 - \frac{v^2}{c^2}}} \to \frac{9c^4/2}{(2\sqrt{c^2})^3/2 (4-\sqrt{c^2})^{1/2}} \to \frac{9v}{2\sqrt{2}}.
\]

\[
= \frac{9}{8\sqrt{2}} < 1.
\]

Presumably, it is \( \gamma \)\(_{\text{min}}\).

Why does the velocity fall as the incoming velocity \( v \) rises? Strange!

Work out Compton effect model

\[
x = \frac{2\hbar}{mc} = \frac{c}{v} - \frac{c}{v_f},
\]

\[
\frac{1}{v_f} = \frac{2\hbar}{mc^2} + \frac{1}{v}.
\]

\[
v_f = \frac{mc^2}{\sqrt{\hbar^2 + mc^2}}.
\]

Momentum gives

\[
\frac{\hbar}{c} = -\frac{\hbar}{c} + \frac{mv}{c} \left( \sqrt{1 - \frac{W^2}{c^2}} \right) - \frac{Wv}{c}.
\]

\[
\frac{mm\sqrt{c}}{\sqrt{1 - \frac{W^2}{c^2}}} = \frac{\hbar}{c} + \frac{\hbar v}{c} = \frac{\hbar}{c^2} \left( \sqrt{2\hbar v + mc^2} + mc^2 \right) \]

\[
\frac{W^2}{c^2} = \frac{\sqrt{2\hbar v}}{mc^2} \left( \sqrt{2\hbar v + mc^2} + mc^2 \right) \]

\[
\frac{W^2}{c^2} = \frac{\sqrt{2\hbar v}}{mc^2} \left( \frac{\sqrt{2\hbar v + mc^2}}{mc^2} \right)^2 - (C)
\]
\[
\frac{1}{(1 - \frac{v^2}{c^2})} = 1 + \frac{\sqrt{\frac{\alpha \gamma (\gamma v + mc^2)}{mc^2 (\gamma v + mc^2)}}^2}{\sqrt{\left[\alpha \gamma (\gamma v + mc^2)\right]^2 + \left[\alpha \gamma (\gamma v + mc^2)\right]^2}}
\]

\[
\frac{w^2}{c^2} = \frac{\left[\alpha \gamma (\gamma v + mc^2)\right]^2}{\left[\alpha \gamma (\gamma v + mc^2)\right]^2 + \left[\alpha \gamma (\gamma v + mc^2)\right]^2}
\]

(Denominator on RHS of (1))

\[
= m^2 c^4 (4 \alpha^2 \gamma^2 + 4 \alpha \gamma mc^2 + mc^4)
\]

\[
+ 4 \alpha^2 \gamma^2 (mc^4 + 4 \alpha \gamma mc^2 + \alpha^2 \gamma^2 c^2)
\]

\[
= m^4 c^8 + 4 \alpha \gamma m^3 c^6 + 8 \alpha \gamma^2 m^2 c^4
\]

\[
+ 8 \alpha^2 \gamma^3 m c^2 + 4 \alpha^2 \gamma^4 c^4
\]

\[
= (mc^2 + \alpha \gamma)^4
\]

\[
+ 2 \alpha \gamma^2 m^2 c^4 + 4 \alpha^2 \gamma^3 m c^2 + 3 \alpha^2 \gamma^4 c^4
\]

\[
= (mc^2 + \alpha \gamma)^4 + 2 \alpha \gamma^2 (mc^2 + \alpha \gamma)^2 + (\alpha \gamma)^4
\]

\[
= \left[(mc^2 + \alpha \gamma)^2 + \alpha^2 \gamma^2\right]^2
\]

\[
\frac{w}{c} = \frac{2 \alpha \gamma (\gamma v + mc^2)}{\left[(mc^2 + \gamma v)^2 + \alpha \gamma^2 \right]^2}
\]

\[
(1 - \frac{w}{c}) = \frac{m^2 c^4}{\left[(mc^2 + \gamma v)^2 + \alpha \gamma^2 \right]^2}
\]