R®LATIVITY THEORY IS INCONSISTENT

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Introduction. The two major areas of
modern physics are relativity theory
and quantum mechanics. But relativity
theory is deterministic and quantum
mechanics is a probabilistic mechanics.
Thus, relativity theory and quantum
mechanics are not compatible. However,
relativity theory has been used with
success for events in the large;
whereas non-relativistic quantum
mechanics has been used for small
-scale events with success. Relativistic
quantum mechanics is not much
developed. Note that some
researchers, including Einstein and
De Broglie, tried to make quantum
mechanics deterministic by invoking "hidden variables." Also, Dirac wrote about the relativistic electron, but little more has been published on relativistic quantum mechanics. Thus, for small-scale events at high speed, physicists use a curious amalgam of relativity theory and quantum mechanics. Physicists have not thought that this amalgam was inconsistent, only that it was not completely true to the world. By inconsistent, we mean that the theory arrives at contradictory results.

This article displays that the conventional amalgam of relativity theory and quantum mechanics is inconsistent.

The main result. Consider an atomic particle of rest mass m grams moving in a straight line at v cm/sec and hit at right angles by a photon of energy hv.
The photon is reflected back along its former path with energy $h\nu'$. The particle moves off in a straight line with velocity $w$ cm/sec at an angle $\phi$ to its former path. See Figure 1. Write $c$ for the velocity of light. Conservation of momentum yields

\[
\frac{h\nu}{c} \mathbf{i} + \frac{mv}{\sqrt{1-v^2/c^2}} \mathbf{f} = -\frac{h\nu'}{c} \mathbf{i} + \frac{mw}{\sqrt{1-w^2/c^2}} \mathbf{n}
\]

--- (1)
\[ \dot{v} \cdot j = 0 \text{ and } \mathbf{v} = \sin(\theta) \mathbf{i} + \cos(\theta) \mathbf{j}. \]

The components of (1) give:

\[ \frac{\mathbf{v} \cdot (v+u)}{c} = \frac{mv}{\sqrt{1-v^2/c^2}} \sin(\theta) \quad (2) \]

and

\[ \frac{mv}{\sqrt{1-v^2/c^2}} = \frac{mv}{\sqrt{1-u^2/c^2}} \cos(\theta) \quad (3) \]

(2) ÷ (3) to eliminate \( v \):

\[ \frac{\frac{\mathbf{v} \cdot (v+u)}{c}}{\left( \frac{mv}{\sqrt{1-v^2/c^2}} \right)} = \tan(\theta) \quad (4) \]

Suppose that the collision is elastic and thus that energy is conserved. Conservation of energy yields:

\[ \hbar v \mathbf{i} + \frac{mc^2}{\sqrt{1-v^2/c^2}} \mathbf{j} \]

\[ = \hbar v \mathbf{i} + \frac{mc^2}{\sqrt{1-u^2/c^2}} \mathbf{j} \]

\[ = \hbar v \mathbf{i} + \frac{mc^2}{\sqrt{1-v^2/c^2}} \mathbf{n} \quad (5) \]
5. The components of (5) give

\[ h(v-v') = \frac{mc^2}{\sqrt{1-v'^2/c^2}} \sin(\theta) \quad \quad (6) \]

\[ \frac{mc^2}{\sqrt{1-v'^2/c^2}} = \frac{mc^2}{\sqrt{1-w'^2/c^2}} \cos(\theta) \quad \quad (7) \]

(6) ÷ (7) to eliminate \( w' \):

\[ \frac{h(v-v')}{\frac{mc^2}{\sqrt{1-v'^2/c^2}}} = \tan(\theta) \quad \quad (8) \]

(4) and (8) give, by eliminating \( \phi \):

\[ \frac{\frac{h}{c}(v+v')}{\left(\frac{mv}{\sqrt{1-v'^2/c^2}}\right)} = \frac{\frac{h}{c}(v-v')}{\left(\frac{mc^2}{\sqrt{1-w'^2/c^2}}\right)} \quad \quad (9) \]

(9) simplifies to

\[ \frac{v}{c} = \frac{(v+v')}{(v-v')} \quad \quad (10) \]

But it is agreed that the re-
coiling photon has lower energy than before, namely, $\nu > \nu' > 0$ --- (11)
in (10) yields
\[
\frac{\nu}{c} < 1;
\]
that is, $\nu > c > 0$ _______ (12)

This contradicts relativity theory which claims that no particle with rest mass $m > 0$ can travel at velocity $\nu > c > 0$, where $c$ is the velocity of light.

The above reasoning neglects acceleration and so is a result in the special theory of relativity. However, if there is acceleration due to gravitational forces, we can consider a very brief interval of time before the impact and a very brief interval of time after the impact. In these small intervals of time, the motion of the atomic particle will be very nearly a pair of straight lines unless the event occurs near a black hole.
Thus, the commonly used amalgam of relativity and quantum mechanics is inconsistent.

Consequences. One might think that a hidden variables theory would reconcile relativity and quantum mechanics by making both deterministic. For details of hidden variables theories, see de Broglie [1] and Einstein, Podolsky, and Rosen [4]; and Wightman [9, pp. 182-193] and the references given there, which appear on Wightman [9, p. 236].

For a popular account of a recent experiment (which contradicts Einstein's theory of relativity) at the University of Geneva by Nicolas Gisin, see Ikonicoff [5].

However, Einstein [3] was at pains to build up relativity theory logically. A more mathematical development of mathematical treatment is found in Dyson [2]. How could such a carefully built theory emerge as inconsistent?
Perhaps relativity theory is inconsistent because the arithmetic of real numbers is inconsistent?

Hilbert announced in 1900 several problems in mathematics research. The second problem was to prove that the axioms of arithmetic are not contradictory, that is, that a finite number of logical steps based upon them can never lead to contradictory results. See Hilbert [6, p. 9]. Thus, far no such proof exists in the published literature. This in spite of all the efforts of Hilbert and his followers in Beweis-theorie (proof theory). For a survey of Hilbert's second problem and the associated literature, see Kreisel [7]. For an earlier survey in German, see Esenin-Volpin [8].

The present author intends to publish several papers in the near future on this problem of the consistency of arithmetic.
REFERENCES

[1] de Broglie, L., La mécanique quantique, restera-t-elle encore indéterministe?


