Congruences associated with FLT.

Consider a prime $p > 3$ and $x \not\equiv 0 \pmod{p}$ and $y \not\equiv 0 \pmod{p}$ such that

$$(x^2 + xy + y^2) \equiv 0 \pmod{p^2}$$

Then $$(x^2 + xy + y^2)^2 \equiv 0 \pmod{p}.$$ Also, if $x \equiv y \pmod{p},$

$$(x-y)(x^3 + xy + y^2) \equiv 0 \pmod{p}.$$ 

ie $$(x^3 \equiv y^3 \pmod{p})$$

So $x^3 \equiv y^3 \pmod{p}.$

But $p = (3n+1)$ or $(3n+2)$ for some integer $n.$

For $p = (3n+2),$ $x^3 \equiv y^3 \pmod{p}.$

But also $x^p \equiv y^p \pmod{p}$

ie $$(x^{(3n+2)}) \equiv (y^{(3n+2)}) \pmod{p}.$$ 

So $x^2 \equiv y^2 \pmod{p},$ also

$$(x^2 \equiv y^2 \pmod{p}) \text{ and } x \equiv y \pmod{p}.$$ 

So $x \equiv y \pmod{p},$ a contradiction.
Congruences associated with FLT.

If \( x \equiv y \pmod{p} \), then
\[ 0 \equiv (x^2 + xy + y^2) \pmod{p} \]
becomes
\[ 3x^2 \equiv 0 \pmod{p} \]
so \( 3 \equiv 0 \pmod{p} \) or \( x^2 \equiv 0 \pmod{p} \) (Thus \( p = 3 \))
which is a contradiction.

Or \( x \equiv 0 \pmod{p} \) which is a contradiction.

For \( p = (3n+1) \), \( x^3 \equiv y^3 \pmod{p} \)
and \( x^1 \equiv y^1 \pmod{p} \) indicate that \( 3n \equiv 3^n \pmod{p} \) and
\[ (3n+1) \equiv y^{3n+1} \pmod{p} \]
and thus \( x \equiv y \pmod{p} \) which leads to one or more of several contradictions (as already shown).

All this shows that Pollacsek's assumptions were invalid and probably all Pollacsek's work must be deemed to be erroneous.
Congruences associated with FLT.

FLT means Fermat's Last Theorem, which Pierre Fermat asserted that he had proven but he did not announce the details of the proof.

Pollaczek's research article is in German and was published early in the 20th century and was entitled "Uber Fermat's Grosse Theorem" (or some such German phrase).

Pollaczek's article is probably referred to in each of several research articles in the interval 1975 to 1995 and thus "Pollaczek" will be in Science Citation Index for each of several years.
Consider $p > 3$ and $x \neq 0 \pmod{p}$ and $y \neq 0 \pmod{p}$ but such that 
\[(x^2 - xey + y^2) = 0 \pmod{p^2}\] and thus 
\[(x^2 - xey + y^2) = 0 \pmod{p}\]

Then \[(x + y)(x^2 - xey + y^2) = 0 \pmod{p}\]
and thus 
\[(x^3 + y^3) = 0 \pmod{p}\]
and thus 
\[x^3 = (y^3) \pmod{p}\]

But \(p = (3n + 1)\) or \(3n + 2)\) where \(n\) is some integer.

For \(p = (3n + 1)\) \(n\) is odd and \(3^n = 3\) \(n\) is odd and \(3^n = 3\) \(n\) is odd and \(3^n = 3\)

But also \(x^p = y^p \pmod{p}\)

\[(x^p) = (y^p) \pmod{p}\]

So \(x^3 = (y^3) \pmod{p}\)

But \(x^3 = (y^3) \pmod{p}\) also.

So \(x = (y) \pmod{p}\) a contradiction.
§ 2

Congruences associated with FLT.

If \( x^2 \equiv y \mod p \), then \( 0 \equiv (x^2 + xy + y^2) \mod p \) becomes \( x^2 \equiv 0 \mod p \); so \( x \equiv 0 \mod p \) which is a contradiction.

For \( p = 3n+1 \), \( x^3 \equiv y \mod p \) and \( x^p \equiv y^p \mod p \) means

\( x^{3n+2} \equiv y^{3n+2} \mod p \) and thus \( x^{3n+2} \equiv (-y)^{3n+2} \mod p \).

But we also have that \( x^{3n} \equiv (-y)^{3n} \mod p \).

So \( x^2 \equiv -y^2 \mod p \).

This is \( (x^2 + y^2) = 0 \mod p \);

but \( (x^2 - xy + y^2) = 0 \mod p \) also.

Thus \( (xy) = 0 \mod p \).

This shows that \( x \equiv 0 \mod p \).