En Polaczek's congruences

This is for first case of FLT, namely, prime $p > 3$ and $x$, $y$, $z$ integers relatively prime to $p$ such that $x^p + y^p + z^p = 0$ and such that $(x^a + xy + y^2) \equiv 0 \pmod{p}$ and similarly for $y$ and $z$ and also for $z$ and $x$.

Fermat's theorem states that $x^{p-1} \equiv 1 \pmod{p}$ and similarly for $y$ and for $z$. Also, $(x+y+z) \equiv 0 \pmod{p}$, so $x^2 \equiv y^2 \pmod{p}$ and $y^2 \equiv z^2 \pmod{p}$ and $z^2 \equiv xy \pmod{p}$.

We show that

$$(x^{p-3} + y^{p-3} + z^{p-3}) \equiv 0 \pmod{p}$$

and

$$(x^{p-6} + y^{p-6} + z^{p-6}) \equiv 0 \pmod{p}$$

and

$$(x^{p-9} + y^{p-9} + z^{p-9}) \equiv 0 \pmod{p}$$
etc.

If \( p = (6n+5) \), then in particular
\[
(y^{(3n+2)} + y^{(3n+2)} + y^{(3n+2)}) \equiv 0 \pmod{p}
\]
also, from \( (6y^{(3n+3)} + y^{(3n+3)} + 8^{(3n+3)}) \equiv 0 \pmod{p} \)
we obtain
\[
(y^{(3n+3)} + 8^{(3n+3)} x^{(3n+3)} + 8^{(3n+3)}) \equiv 0 \pmod{p}
\]
this is equivalent to
\[
(x^{(3n+3)} + x^{(3n+3)} y^{(3n+3)} + 8^{(3n+3)}) \equiv 0 \pmod{p}
\]
but this is
\[
(x^{(3n+1)} + y^{(3n+1)} + 8^{(3n+1)}) \equiv 0 \pmod{p}
\]
multiply by \( x \) and have
\[
(x y^{(3n+1)} + x y^{(3n+1)} + 8^{(3n+1)}) = x^{(3n+2)}
\]
\[
(3n+2) \equiv (y^{(3n+2)} + 8^{(3n+2)}) \equiv 0 \pmod{p}
\]
thus
\[(x - y)^{3n+1} \equiv (z - x)^{3n+1} \pmod{p}\]

Square both sides:
\[(x - y)^{2} (6n+2) \equiv (z - x)^{2} (6n+2) \pmod{p}\]

Multiply both sides by \(y^2\):
\[(x - y)^{2} y (6n+4) \equiv (z - x)^{2} y (6n+4) \pmod{p}\]

Use Fermat’s Little Theorem:
\[(x - y)^{2} \equiv (z - x)^{2} \pmod{p}\]

Take square roots of both sides:
\[(x - y) \equiv (z - x) \pmod{p}\]

Or
\[(x - y) \equiv - (z - x) \pmod{p}\]

Using first congruence of the two congruences:
\[(x^2 - y^2) \equiv (y^2 - x^2y) \pmod{p}\]

This becomes:
\[ 0 \equiv \nabla \text{show earlier} \equiv 0 \]

\[ (x^3 + y^3 + 3y) \equiv (x^3 + 3y) \mod p \]

or \[ 3y + 3y \equiv 0 \mod p \]

which is a contradiction.

Using second congruence of the two congruences:

\[ (x^3 - y^3) \equiv -(x^3 - y^3) \mod p \]

This is \[ x^3 - y^3 \equiv 0 \mod p \].

If we use Pollaczek's extras \[ x \equiv y \mod p \] and \[ y \equiv z \mod p \] and \[ z \equiv x \mod p \], we have a contradiction here also.

Thus there was an error in the assumptions. This error could be that \[ (x^3 + y^3 + \beta^2) \equiv 0 \]

and \[ (y^3 + x^3 + \beta^2) \equiv 0 \mod p \] plus \[ (y^3 + z^3 + \beta^2) \equiv 0 \mod p \] plus \[ (\beta^3 + 3x^2 + x^2) \equiv 0 \mod p \] etc.