To show that a cube of a positive integer cannot be the sum of the cubes of 3 non-zero positive integers.

Let \( u^3 = x^3 + y^3 + z^3 \) where \( u, x, y, z \) are positive integers.

\[
u^3 - x^3 = y^3 + z^3.
\]

\[
(u-x)(u^2 + ux + x^2) = (y+z)(y^2 - yz + z^2)
\]

\[
(u-x)[(u-x)^2 + 3ux] = (y+z)[(y+z)^2 - 3yz]
\]

Let \( u-x = \left(\frac{r}{5}\right)(y+z) \) where \( r, s \) are non-zero integers. Then

\[
\left(\frac{r}{5}\right)^3 (y+z)^3 + (3ux)\left(\frac{r}{5}\right)(y+z)
\]

\[
= (y+z)^3 - (3yz)(y+z).
\]

This is changed by taking out the non-factor \( (y+z) \). We obtain

\[
\left(\frac{r}{5}\right)^3 (y+z)^2 + (3ux)\left(\frac{r}{5}\right) = \left(\frac{r}{5}\right)
\]

\[
- \left[ (y+z)^2 - 3yz \right] = 0.
\]

The 3 roots for \( \left(\frac{r}{5}\right) \) sum to zero.
In our extra conditions, we show that only positive values for \( \varepsilon \) are acceptable.

\[
\alpha + \beta + \gamma = 0, \quad \text{where} \quad \frac{r}{\varepsilon} = \alpha \beta \gamma.
\]

\[
\alpha \beta + \beta \gamma + \gamma \alpha = \frac{3 \mu x e}{(y + z)^2} > 0 \quad \text{and real}
\]

\[
\beta \varepsilon - \alpha^2 = \frac{3 \mu x e}{(y + z)^2} > 0 \quad \text{and real}
\]

\[
\gamma \alpha - \beta^2 = \alpha \beta - \varepsilon^2
\]

So \( \alpha, \beta, \gamma \) are all real because \( x, \text{real}, \beta = (a + bi), a, \text{real} \), \( \beta > 0 \).

This can only mean that all real to the condition \( \alpha + \beta + \gamma = 0 \). This is contrary to the condition \( \alpha + \beta + \gamma = 0 \).

Thus, no such cube as the sum of three cubes can exist.

I think that \( k^3 + 3mk^n + n = 0 \) has all three roots real if \( m, n \) real and \( (m^3 + n^2) > 0 \).

For \( m = \frac{ux}{(y + z)^2} \), \( n = -\frac{(y^3 + x^3 + z^2)}{(y + z)^2} \), this is

\[
\frac{ux^3 + (y^3 + z^2)^2}{4} > 0 \quad \text{i.e.} \quad \frac{ux^3 + 4(m^3 - v^3)^2}{4} > 0.
\]

\( (u^3 + x^3)^2 > 0 \), true: so are 3 real roots, but are

no roots only one possible: they make some

Exactly 3 solutions? Can't be same \((u^3 + x^3)^2 \)