FLT for negative exponents.

This includes the pythagorean equation analogue with negative exponents.

Let $x, y, z$ be integers with $gcd(x)$ and let $n$ be an integer > 1 and let

$$\frac{1}{x^n} = \frac{1}{y^n} + \frac{1}{z^n}$$

So

$$y^n \cdot z^n = x^n (y^n + z^n)$$

(1)

Let $D = gcd(y, z)$, $y = D \cdot y_1$, $z = D \cdot z_1$, where $y_1, z_1$ are integers. Then divide (1) by $D^n$:

$$d^n \cdot z^n = x^n (d^n + z_1^n)$$

This is

$$d^n \cdot d_1^n \cdot z^n = x^n (d_1^n + d_2^n)$$

(2)

But $gcd(D, x) = gcd(gcd(D, y), z) = gcd(x, y)$

= 1. So let $d_2 = s \cdot x$, $s$ positive integer.

because $gcd(s, x) > 1$, then

$$d_2^n \cdot z^n = (d_1^n + d_2^n)$$

(3)

So $d_1^n (s \cdot D^n - 1) = d_2^n x$, showing that

$$d_1^n | d_2^n$$

So $d_1^n = 1$, $s \cdot (3)$ is
\[ f_{2n} = 1 + d^2 \]

For \( n > 2 \), \( d_1 < 8D < (1+d_3) \).

So \( S'D \) is not an integer.

This contradiction proves such \( x, y, z \) and \( n \) do not exist.

"On the analogues of the Pythagorean equation and of Fermat's equation for negative exponents,"