What is the characteristic polynomial of a signal flow graph?

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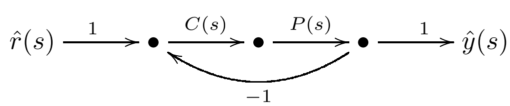
Warmup

- Consider the unity gain feedback loop as a signal flow graph:



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- As we teach our undergraduates, the four transfer functions

$$\frac{1}{1+P(s)C(s)}, \quad \frac{P(s)}{1+P(s)C(s)}, \quad \frac{C(s)}{1+P(s)C(s)}, \quad \frac{P(s)C(s)}{1+P(s)C(s)}$$

are analytic in the CRHP if and only if the polynomial

$$P_{\mathcal{S},\mathcal{G}}(s) = D_P(s)D_C(s) + N_P(s)N_C(s)$$

is Hurwitz, with $P(s) = \frac{N_P(s)}{D_P(s)}$ and $C(s) = \frac{N_C(s)}{D_C(s)}$ coprime.

- The punchline is that $P_{\mathcal{S},\mathcal{G}}(s)$ is obtained by "looking at the graph," and requires no polynomial factorisation, etc.

Problem

- Let $(\mathcal{S}, \mathcal{G})$ be a signal flow graph comprised of ***signals*** or ***nodes*** $\mathcal{S}$ and branches with (proper) gains specified by $\mathcal{G}$.
- For example

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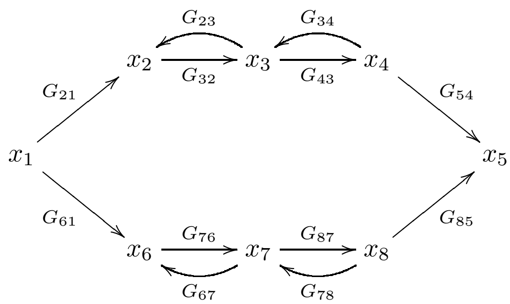

- Using Mason's Rule, compute the transfer functions $T_{ij}(s)$ from node i to node j, $i, j \in \{1, \ldots, n\}$.
- $(\mathcal{S}, \mathcal{G})$ is ***BIBO stable*** when each $T_{ij}(s) \in \mathrm{RH}_\infty$, $i, j \in \{1, \ldots, n\}$.
- Each $T_{ij}(s)$ is proper if and only if graph determinant $\Delta_{\mathcal{S},\mathcal{G}}(s)$ is biproper $\Longrightarrow$ need condition for $T_{ij}(s)$ to be analytic in CRHP.

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Problem: *Is there a polynomial $P_{\mathcal{S},\mathcal{G}}(s)$, constructed from the topology of $(\mathcal{S}, \mathcal{G})$, which is Hurwitz if and only if the transfer functions $T_{ij}(s)$, $i, j \in \{1, \ldots, n\}$, are analytic in the CRHP?*

- *Note:* The polynomial matrix approach gives an algorithm for solving any given problem, but generally polynomials will need to be factored.