Notes on energy shaping

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The problem

- *Roughly speaking:* Using feedback, turn a mechanical system into another mechanical system with desired properties, e.g., stability.
- *Less roughly:*
  Given: An open-loop simple mechanical control system $\Sigma_{\mathsf{ol}} = (\mathsf{Q}, \mathbb{G}_{\mathsf{ol}}, V_{\mathsf{ol}}, \mathscr{F} = \{F^1, \dots, F^m\})$ with control equations

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$$\overset{\mathbb{G}_{\mathrm{ol}}}{\nabla}_{\gamma'(t)}\gamma'(t) = -\mathbb{G}_{\mathsf{ol}}^{\sharp} \circ \boldsymbol{d}V_{\mathsf{ol}}(\gamma(t)) + \sum_{a=1}^{m} u^a(t)\mathbb{G}_{\mathsf{ol}}^{\sharp} \circ F^a(\gamma(t))$$

$$\text{i.e., } \Big(\ddot{q}^i + \overset{\mathbb{G}_{\mathrm{ol}}}{\Gamma}{}^i_{jk}\dot{q}^j\dot{q}^k = -\mathbb{G}_{\mathrm{ol}}^{ij}\frac{\partial V_{\mathrm{ol}}}{\partial q^j} + \sum_{a=1}^{m} u^a \mathbb{G}_{\mathrm{ol}}^{ij} F_j^a\Big).$$

Find: Feedback controls $u_{\mathsf{shp}} \colon \mathsf{TQ} \to \mathbb{R}^m$ such that the closed-loop system is a forced simple mechanical system $\Sigma_{\mathsf{cl}} = (\mathsf{Q}, \mathbb{G}_{\mathsf{cl}}, V_{\mathsf{cl}}, F_{\mathsf{cl}})$ with governing equations

$$\overset{\mathbb{G}_{\mathrm{cl}}}{\nabla}_{\gamma'(t)}\gamma'(t) = -\mathbb{G}_{\mathsf{cl}}^{\sharp} \circ \boldsymbol{d}V_{\mathsf{cl}}(\gamma(t)) + \mathbb{G}_{\mathsf{cl}}^{\sharp} \circ F_{\mathsf{cl}}(\gamma'(t)).$$

- *Form of F_{cl}:* $F_{\mathsf{cl}} = F_{\mathsf{cl,diss}} + F_{\mathsf{cl,gyr,1}} + F_{\mathsf{cl,gyr,2}}$ where
  1. $F_{\mathsf{cl,diss}}$ is a dissipative force,
  2. $F_{\mathsf{cl,gyr,1}}$ is a linear gyroscopic force, and
  3. $F_{\mathsf{cl,gyr,2}}$ is a quadratic gyroscopic force.
- *Recall:*

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  1. A **linear gyroscopic force** is of the form $F_{\mathsf{gyr,1}}(v_q) = -B^\flat_{\mathsf{gyr,1}}(v_q)$ where $B_{\mathsf{gyr,1}}$ is a skew-symmetric $(0,2)$-tensor field.
  2. A **quadratic gyroscopic force** is of the form $\langle F_{\mathsf{gyr,2}}(v_q); w_q\rangle = B_{\mathsf{gyr,2}}(w_q, v_q, v_q)$ where $B_{\mathsf{gyr,2}}$ is a $(0,3)$-tensor field satisfying $B_{\mathsf{gyr,2}}(u_q, v_q, w_q) = -B_{\mathsf{gyr,2}}(v_q, u_q, w_q)$ (denote $F_{\mathsf{gyr,2}}(v_q) = B^\flat_{\mathsf{gyr,2}}(v_q)$).

- *Assumed procedure:*
  1. Find closed-loop kinetic energy/quadratic gyroscopic force:
$$\mathbb{G}^\sharp_{\mathsf{ol}} \circ F_{\mathsf{kin}}(\gamma'(t)) = \overset{\mathbb{G}_{\mathrm{cl}}}{\nabla}_{\gamma'(t)}\gamma'(t) + \mathbb{G}^\sharp_{\mathsf{cl}} \circ B^\flat_{\mathsf{cl,gyr,2}}(\gamma'(t)) - \overset{\mathbb{G}_{\mathrm{ol}}}{\nabla}_{\gamma'(t)}\gamma'(t)$$
  2. Find closed-loop potential energy:

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$$F_{\mathsf{pot}}(\gamma(t)) = \underbrace{\mathbb{G}^\flat_{\mathsf{ol}} \circ \mathbb{G}^\sharp_{\mathsf{cl}}}_{\Lambda_{\mathsf{cl}}} \circ \boldsymbol{d}V_{\mathsf{cl}}(\gamma(t)) - \boldsymbol{d}V_{\mathsf{ol}}(\gamma(t)).$$
  3. Find closed-loop control:
$$\sum_{a=1}^m u^a_{\mathsf{shp}}(v_q)\mathbb{G}^\sharp_{\mathsf{ol}} \circ F^a(q) = -F_{\mathsf{kin}}(v_q) - F_{\mathsf{pot}}(q).$$
- *Today:* Ignore dissipative and linear gyroscopic forces.

Main problem

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What are the possible closed loop energies,

$$E_{\mathrm{cl}}(v_q) = \tfrac{1}{2}\mathbb{G}_{\mathrm{cl}}(v_q) + V_{\mathrm{cl}}(q)?$$

Partial literature review and comments

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- Potential shaping:
  1. M. Takegaki and S. Arimoto, "A new feedback method for dynamic control of manipulators," *Trans. ASME Ser. G J. Dynamic Systems Measurement Control*, vol. 103, no. 2, pp. 119–125, 1981.
  2. A. J. van der Schaft, "Stabilization of Hamiltonian systems," *Nonlinear Anal. TMA*, vol. 10, no. 10, pp. 1021–1035, 1986.
  3. A. M. Bloch, P. S. Krishnaprasad, J. E. Marsden, and G. Sánchez de Alvarez, "Stabilization of rigid body dynamics by internal and external torques," *Automatica—J. IFAC*, vol. 28, no. 4, pp. 745–756, 1992.

- Hamiltonian approach (IDA-PBC):
  4. R. Ortega, M. W. Spong, F. Gómez-Estern, and G. Blankenstein, "Stabilization of a class of underactuated mechanical systems via interconnection and damping assignment," *IEEE Trans. Automat. Control*, vol. 47, no. 8, pp. 1218–1233, 2002.
- Lagrangian approach with symmetry:

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  5. A. M. Bloch, N. E. Leonard, and J. E. Marsden, "Controlled Lagrangians and the stabilization of mechanical systems. I. The first matching theorem," *IEEE Trans. Automat. Control*, vol. 45, no. 12, pp. 2253–2270, 2000.
  6. A. M. Bloch, D. E. Chang, N. E. Leonard, and J. E. Marsden, "Controlled Lagrangians and the stabilization of mechanical systems. II. Potential shaping," *IEEE Trans. Automat. Control*, vol. 46, no. 10, pp. 1556–1571, 2001.

- Equivalence of Lagrangian and Hamiltonian setting:
  7. D. E. Chang, A. M. Bloch, N. E. Leonard, J. E. Marsden, and C. A. Woolsey, "The equivalence of controlled Lagrangian and controlled Hamiltonian systems," *ESAIM Contrôle Optim. Calc. Var.*, vol. 8, pp. 393–422, 2002.
  8. G. Blankenstein, R. Ortega, and A. J. van der Schaft, "The matching conditions of controlled Lagrangians and IDA-passivity based control," *Internat. J. Control*, vol. 75, no. 9, pp. 645–665, 2002.

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- Geometric formulation and integrability:
  9. D. R. Auckly and L. V. Kapitanski, "On the λ-equations for matching control laws," *SIAM J. Control Optim.*, vol. 41, no. 5, pp. 1372–1388, 2002.
  10. D. R. Auckly, L. V. Kapitanski, and W. White, "Control of nonlinear underactuated systems," *Comm. Pure Appl. Math.*, vol. 53, no. 3, pp. 354–369, 2000.

- Extension to general Lagrangians:
  11. J. Hamberg, "General matching conditions in the theory of controlled Lagrangians," in *Proceedings of the 38th IEEE CDC*. Phoenix, AZ: IEEE, Dec. 1999, pp. 2519–2523.
  12. J. Hamberg, "Simplified conditions for matching and for generalized matching in the theory of controlled Lagrangians," in *Proceedings of the 2000 American Control Conference*, Chicago, IL, June 2000, pp. 3918–3923.

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- Extension to nonholonomic systems:
  13. D. V. Zenkov, "Matching and stabilization of the unicycle with rider," in *Proc. IFAC Workshop on Lagrangian and Hamiltonian Methods in Nonlinear Control*, Princeton, NJ, March 2000, pp. 187–188.
- Linear systems:
  14. D. V. Zenkov, "Matching and stabilization of linear mechanical systems," in *Proceedings of MTNS 2002*, South Bend, IN, Aug. 2002.

- *Major limitation of applicability:* For stabilization, only works for systems that are linearly stabilizable, i.e., doesn't work for "hard" systems (i.e., requiring discontinuous feedback).

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- *Advantages of ideas:*
  1. Allows user to employ mechanical intuition concerning the closed-loop design.
  2. Increased performance (particularly basic of attraction) over some other techniques.

Notes on potential energy shaping

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- *Assume that the closed-loop kinetic energy/quadratic gyroscopic force have been determined.*
- Let $\mathcal{F}$ be the codistribution generated by the input forces $\{F^1, \dots, F^m\}$.
- Let $\mathcal{F}_{\mathsf{cl}} = \Lambda_{\mathsf{cl}}^{-1}(\mathcal{F})$.
- Let $\mathcal{F}_{\mathsf{cl}}^{(\infty)}$ be the largest integrable codistribution contained in $\mathcal{F}_{\mathsf{cl}}$.
- Assume that $\mathcal{F}_{\mathsf{cl}}$ and $\mathcal{F}_{\mathsf{cl}}^{(\infty)}$ have constant rank.
- $C^\infty(\mathsf{Q})$: set of C^∞-functions on Q
  $C^\infty(\mathsf{Q})_{\mathcal{F}_{\mathsf{cl}}^{(\infty)}}$: subset of $C^\infty(\mathsf{Q})$ for which $\boldsymbol{d}f(q) \in \mathcal{F}_{\mathsf{cl},q}^{(\infty)}$
  (same as subset of $C^\infty(\mathsf{Q})$ for which $\boldsymbol{d}f(q) \in \mathcal{F}_{\mathsf{cl},q}$)

Proposition 1 *The set of closed-loop potentials is an affine subspace, possibly empty, of $C^\infty(\mathsf{Q})$ whose linear part is $C^\infty(\mathsf{Q})_{\mathcal{F}_{\mathsf{cl}}^{(\infty)}}$.*

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- *Discussion:*
  1. The set of closed-loop potentials could be empty, even if $C^\infty(\mathsf{Q})_{\mathcal{F}_{\mathsf{cl}}^{(\infty)}}$ is big (*think $Ax = b$*). The testing of this involves an integrability condition that seems to have not been explored yet.
  2. Generically we expect $\mathcal{F}_{\mathsf{cl},q}^{(\infty)} = 0_q$.
     Punchline: Even if $\mathcal{F}^{(\infty)}$ (the codistribution for classical potential shaping) is big, the set of closed-loop potentials could be small (empty, or consisting of one solution).
  3. Many of the examples in the literature have $\operatorname{codim}(\mathcal{F}_q) = 1$. In these cases, $\mathcal{F}_{\mathsf{cl}}^{(\infty)} = \mathcal{F}_{\mathsf{cl}}$, meaning that there are lots of closed-loop potentials, if there is one.

An affine connection formulation of kinetic energy shaping + quadratic gyroscopic force

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- For a general affine connection ∇ and Riemannian metric $\mathbb{G}$ with its Levi-Civita connection $\overset{\mathbb{G}}{\nabla}$, define a $(0,3)$-tensor field $D_{\nabla,\mathbb{G}}$ by

$$\mathbb{G}(\nabla_X Y, Z) = \mathbb{G}(\overset{\mathbb{G}}{\nabla}_X Y, Z) + D_{\nabla,\mathbb{G}}(Z, X, Y).$$

- For a $(0,k)$-tensor A on V, define a symmetric $(0,k)$-tensor $\mathrm{Sym}(A)$ by

$$\mathrm{Sym}(A)(v_1, \dots, v_k) = \frac{1}{k!} \sum_{\sigma \in S_k} A(v_{\sigma(1)}, \dots, v_{\sigma(k)}).$$

- Similarly define a skew-symmetric $(0,k)$-tensor $\mathrm{Alt}(A)$ by

$$\mathrm{Alt}(A)(v_1, \dots, v_k) = \frac{1}{k!} \sum_{\sigma \in S_k} (-1)^{\mathrm{sgn}(\sigma)} A(v_{\sigma(1)}, \dots, v_{\sigma(k)}).$$

- Think of Sym and Alt as linear maps from $\mathrm{T}^0_k(\mathsf{V})$ to $\mathrm{T}^0_k(\mathsf{V})$.

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- A $(0,3)$-tensor A is **gyroscopic** if $A(u,v,w) = -A(v,u,w)$
  is **torsional** if $A(u,v,w) = -A(u,w,v)$
  $\mathrm{Gyr}(\mathsf{V})$: gyroscopic tensors
  $\mathrm{Tor}(\mathsf{V})$: torsional tensors

- For a Riemannian metric $\mathbb{G}$, define $\mathrm{KE}_{\mathbb{G}} \colon \mathsf{TQ} \to \mathbb{R}$ by $\mathrm{KE}_{\mathbb{G}}(v_q) = \frac{1}{2}\mathbb{G}(v_q, v_q)$.

- An affine connection ∇ is **$\mathbb{G}$-energy-preserving** if $\mathscr{L}_{\gamma''(t)}\mathrm{KE}_{\mathbb{G}}(\gamma'(t)) = 0$ for every geodesic γ of ∇.

Proposition 2 *The following are equivalent:*

(i) ∇ *is* $\mathbb{G}$*-energy preserving;*

(ii) $\nabla\mathbb{G} \in \Gamma^\infty(\ker(\mathrm{Sym}))$*;*

(iii) $D_{\nabla,\mathbb{G}} \in \Gamma^\infty(\ker(\mathrm{Sym}))$*;*

(iv) *there exists tensor fields* $\Omega_{\nabla,\mathbb{G}} \in \Gamma^\infty(\mathrm{T}\bigwedge^3(\mathsf{TQ}))$,
$B_{\nabla,\mathbb{G}} \in \Gamma^\infty((\mathrm{Gyr}(\mathsf{TQ}) \cap \ker(\mathrm{Alt})))$*, and*
$\hat{T}_{\nabla,\mathbb{G}} \in \Gamma^\infty((\mathrm{Tor}(\mathsf{TQ}) \cap \ker(\mathrm{Alt})))$ *such that*

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$$\mathbb{G}(\nabla_X Y, Z) = \mathbb{G}(\overset{\mathbb{G}}{\nabla}_X Y, Z) + B_{\nabla,\mathbb{G}}(Z, X, Y) + \hat{T}_{\nabla,\mathbb{G}}(Z, X, Y) + \Omega_{\nabla,\mathbb{G}}(Z, X, Y),$$

for all $X, Y, Z \in \Gamma^\infty(\mathsf{TQ})$.

- *Discussion:*

1. If B is a gyroscopic tensor field, then there exists a unique torsion-free affine connection ∇ such that

$$\mathbb{G}(\nabla_X X, Y) = \mathbb{G}(\overset{\mathbb{G}}{\nabla}_X X, Y) + B(Y, X, X)$$

for all $X, Y \in \Gamma^\infty(\mathsf{TQ})$.
Explicitly, ∇ is defined by

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$$\mathbb{G}(\nabla_X Y, Z) = \mathbb{G}(\overset{\mathbb{G}}{\nabla}_X Y, Z) + B_{\nabla,\mathbb{G}}(Z, X, Y),$$

where $B_{\nabla,\mathbb{G}} = B - \mathrm{Alt}(B)$.

2. Changes the kinetic energy/quadratic gyroscopic force determination into a purely affine connection problem:

   Find a Riemannian metric $\mathbb{G}_{\mathsf{cl}}$ and a $\mathbb{G}_{\mathsf{cl}}$-energy preserving connection $\overset{\mathrm{cl}}{\nabla}$ such that $\overset{\mathrm{cl}}{\nabla}_{\gamma'(t)}\gamma'(t) - \overset{\mathbb{G}_{\mathrm{ol}}}{\nabla}_{\gamma'(t)}\gamma'(t) \in \mathbb{G}_{\mathsf{ol}}^\sharp(\mathcal{F}_{\gamma(t)})$.

Other results in the paper

- Put the energy shaping problem in a framework where the integrability theory of Spencer, Goldschmidt, Quillen, Serre, et al. can be applied.
- Define

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$$P_{\mathsf{PS}}(V_{\mathsf{ol}})_q = \{j^1 F_{\mathsf{cl}}(q) | \boldsymbol{d}_1(j^1 F_{\mathsf{cl}}(q)) = -\boldsymbol{d}(\Lambda_{\mathsf{cl}} \circ \boldsymbol{d}V_{\mathsf{ol}})(q)\}.$$

Proposition 3 *Suppose that $H^1(\mathsf{Q}) = 0$. Then a function V_{cl} is a possible closed-loop potential function if and only if $\boldsymbol{d}V_{\mathrm{cl}} = F_{\mathrm{cl}} + \Lambda_{\mathrm{cl}} \circ \boldsymbol{d}V_{\mathrm{ol}}$ where F_{cl} is a section of $\mathcal{F}_{\mathrm{cl}}$ having the property that $j^1 F_{\mathrm{cl}}$ takes values in $P_{\mathrm{PS}}(V_{\mathrm{ol}})$.*

- Define

$$\mathrm{ES}(\mathsf{Q}) = \Sigma_2^+(\mathsf{TQ}) \times (\mathrm{Gyr}(\mathsf{TQ}) \cap \ker(\mathrm{Alt}))$$

and

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$$\begin{aligned} P_{\mathsf{KS}}(\mathbb{G}_{\mathsf{ol}})_q = \big\{ & (j^1\mathbb{G}(q), j^1 B(q)) \in J^1(\mathrm{ES}(\mathsf{Q})) \big| \\ & (\mathrm{LC}(j^1\mathbb{G}(q)) - \mathrm{LC}(j^1\mathbb{G}_{\mathsf{ol}}(q)) + \mathbb{G}^\sharp B \in \mathbb{G}_{\mathsf{ol}}^\sharp(\mathcal{F} \otimes \mathrm{TS}^2(\mathsf{TQ}))\}. \end{aligned}$$

Proposition 4 *A Riemannian metric $\mathbb{G}_{\mathrm{cl}}$ and a gyroscopic tensor field B_{cl} solve the kinetic energy/quadratic gyroscopic problem if and only if the 1-jet of the section $q \mapsto (\mathbb{G}_{\mathrm{cl}}(q), B_{\mathrm{cl}}(q))$ takes values in $P_{\mathrm{KS}}(\mathbb{G}_{\mathrm{ol}})$.*

Things remaining undone in the energy shaping problem

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- Almost everything. For example:
  1. integrability for energy-preserving affine connection problem—what is the form of the set of closed-loop metrics?
  2. integrability for potential shaping problem—sufficient/necessary conditions for closed-loop metric that ensure solution to potential shaping problem.
  3. Is this method implementable, or just interesting?