Preface

These notes deal primarily with the subject of Lagrangian mechanics. Matters related to mechanics are the dynamics and control of mechanical systems. While dynamics of Lagrangian systems is a generally well-founded field, control for Lagrangian systems has less of a history. In consequence, the control theory we discuss here is quite elementary, and does not really touch upon some of the really challenging aspects of the subject. However, it is hoped that it will serve to give a flavour of the subject so that people can see if the area is one which they’d like to pursue.

Our presentation begins in Chapter 1 with a very general axiomatic treatment of basic Newtonian mechanics. In this chapter we will arrive at some conclusions you may already know about from your previous experience, but we will also very likely touch upon some things which you had not previously dealt with, and certainly the presentation is more general and abstract than in a first-time dynamics course. While none of the material in this chapter is technically hard, the abstraction may be off-putting to some. The hope, however, is that at the end of the day, the generality will bring into focus and demystify some basic facts about the dynamics of particles and rigid bodies. As far as we know, this is the first thoroughly Galilean treatment of rigid body dynamics, although Galilean particle mechanics is well-understood.

Lagrangian mechanics is introduced in Chapter 2. When instigating a treatment of Lagrangian mechanics at a not quite introductory level, one has a difficult choice to make; does one use differentiable manifolds or not? The choice made here runs down the middle of the usual, “No, it is far too machinery,” and, “Yes, the unity of the differential geometric approach is exquisite.” The basic concepts associated with differential geometry are introduced in a rather pragmatic manner. The approach would not be one recommended in a course on the subject, but here serves to motivate the need for using the generality, while providing some idea of the concepts involved. Fortunately, at this level, not overly many concepts are needed; mainly the notion of a coordinate chart, the notion of a vector field, and the notion of a one-form. After the necessary differential geometric introductions are made, it is very easy to talk about basic mechanics. Indeed, it is possible that the extra time needed to understand the differential geometry is more than made up for when one gets to looking at the basic concepts of Lagrangian mechanics. All of the principal players in Lagrangian mechanics are simple differential geometric objects. Special attention is given to that class of Lagrangian systems referred to as “simple.” These systems are the ones most commonly encountered in physical applications, and so are deserving of special treatment. What’s more, they possess an enormous amount of structure, although this is barely touched upon here. Also in Chapter 2 we talk about forces and constraints. To talk about control for Lagrangian systems, we must have at hand the notion of a force. We give special attention to the notion of a dissipative force, as this is often the predominant effect which is unmodelled in a purely Lagrangian system. Constraints are also prevalent in many application areas, and so demand attention. Unfortunately, the handling of constraints in the literature is often excessively complicated. We try to make things as simple as possible, as the ideas indeed are not all that complicated. While we do not intend these notes to be a detailed description of Hamiltonian mechanics, we do briefly discuss the link between Lagrangian Hamiltonian mechanics in Section 2.9. The final topic of discussion in Chapter 2 is the matter of symmetries. We give a Noetherian treatment.

Once one uses the material of Chapter 2 to obtain equations of motion, one would like to be able to say something about how solutions to the equations behave. This is the subject of Chapter 3. After discussing the matter of existence of solutions to the Euler-Lagrange equations (a matter which deserves some discussion), we talk about the simplest part of Lagrangian dynamics, dynamics near equilibria. The notion of a linear Lagrangian system and a linearisation of a nonlinear system are presented, and the stability properties of linear Lagrangian systems are explored. The behaviour is non-generic, and so deserves a treatment distinct from that of general linear systems. When one understands linear systems, it is then possible to discuss stability for nonlinear equilibria. The subtle relationship between the stability of the linearisation and the stability of the nonlinear system is the topic of Section 3.2. While a general discussion the dynamics of Lagrangian systems with forces is not realistic, the important class of systems with dissipative forces admits a useful discussion; it is given in Section 3.5. The dynamics of a rigid body is singled out for detailed attention in Section 3.6. General remarks about simple mechanical systems with no potential energy are also given. These systems are important as they are extremely structure, yet also very challenging. Very little is really known about the dynamics of systems with constraints. In Section 3.8 we make a few simple remarks on such systems.

In Chapter 4 we deliver our abbreviated discussion of control theory in a Lagrangian setting. After some generalities, we talk about “robotic control systems,” a generalisation of the kind of system one might find on a shop floor, doing simple tasks. For systems of this type, intuitive control is possible, since all degrees of freedom are actuated. For underactuated systems, a first step towards control is to look at equilibrium points and linearise. In Section 4.4 we look at the special control structure of linearised Lagrangian systems, paying special attention to the controllability of the linearisation. For systems where linearisations fail to capture the salient features of the control system, one is forced to look at nonlinear control. This is quite challenging, and we give a terse introduction, and pointers to the literature, in Section 4.5.

Please pass on comments and errors, no matter how trivial. Thank you.

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