INSTRUCTIONS

• This test is 90 MINUTES in length and consists of 4 questions.

• Answer all questions, writing clearly in the space provided. If you need more room, continue to answer on the back of the previous page, providing clear directions to the marker.

• SHOW ALL YOUR WORK, clearly and in order, if you wish to receive full credit.

• No textbook, lecture note, calculator, computer, or other aid, is allowed.

• Good luck!

FOR MARKER’S USE ONLY

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1. Consider the system of linear equations given by:

\[
\begin{align*}
    x_1 + 2x_2 - x_3 &= 4, \\
    2x_1 + x_2 &= 2, \\
    -3x_1 - x_2 + x_3 &= 2,
\end{align*}
\]

where we wish to solve for the triple \((x_1, x_2, x_3)\) of real numbers.

(a) Write the **augmented matrix** for this system. (5 pts)

(b) Transform the augmented matrix to row-echelon form using a sequence of elementary row operations (clearly indicate which operation you perform at each step). (5 pts)

(c) Find the set of all solutions of the system of linear equations by applying back-substitution to the system resulting from step (b). (5 pts)
2. (10 pts) Consider the real vector space \((\mathbb{R}^3, +, \cdot)\), and consider the following vectors in \(\mathbb{R}^3\):

\[
v_1 = (1, 0, 1), \quad v_2 = (2, 2, 2), \quad v_3 = (-1, 0, 0).
\]

Determine whether or not \(\{v_1, v_2, v_3\}\) is a generating set for \(\mathbb{R}^3\).
3. Consider again the real vector space $(\mathbb{R}^3, +, \cdot)$, and let $W$ be the subset of $\mathbb{R}^3$ consisting of all triples $(x, y, z)$ of real numbers for which $2x - z = 0$, i.e.

$$W = \{(x, y, z) \in \mathbb{R}^3 \mid 2x - z = 0\}.$$ 

$W$ is a vector subspace of $\mathbb{R}^3$ and hence is itself a real vector space.

(a) **Find a basis for** $W$. (10pts)

(b) **Using (a), determine the dimension of** $W$. (2pts)
4. (8 pts) Let \((V, +, \cdot)\) be a real vector space. Let \(v_1, v_2\) be two distinct elements of \(V\), and assume \(\{v_1, v_2\}\) is a generating set for \(V\). Let now \(v_3 = v_1 + v_2\) and \(v_4 = v_1 - v_2\). Show that \(\{v_3, v_4\}\) is also a generating set for \(V\).

bonus: What can you say about the dimension of \(V\)?