

Problem Set #11

Due: by Monday, April 6, 2009 at 5 pm.

1. (a) Give an example of an operator on \mathbb{C}^4 whose characteristic and minimal polynomials both equal $z(z-1)^2(z-3)$.
(b) Give an example of an operator on \mathbb{C}^4 whose characteristic polynomial equals $z(z-1)^2(z-3)$ and whose minimal polynomial equals $z(z-1)(z-3)$.

2. Define $N \in \text{End}(\mathbb{C}^5)$ by $N(x_1, x_2, x_3, x_4, x_5) = (2x_2, 3x_3, -x_4, 4x_5, 0)$. Find a square root of $I + N$.

3. Consider $T \in \text{End}(V)$.
(a) Suppose T is invertible. Prove that there exists a polynomial $f \in \mathbb{K}[t]$ such that $T^{-1} = f(T)$. (Hint: use the minimal polynomial of T .)
(b) Suppose that $0 \neq v \in V$ and g is the monic polynomial of smallest degree such that $g(T)v = 0$. Prove that g divides the minimal polynomial of T .