

REVIEW

Exam: Monday, April 13 from 9:00am to 12:00pm in GRA main floor.

The exam consists of eight questions each worth 10 points. It covers material the entire semester but emphasizes the second half. Here is a list of topics covered during the semester.

- VECTOR SPACES: axioms, subspaces, examples, direct sums
- BASES: linear combinations, span, linear independence, dimension, existence of bases
- LINEAR MAPS: definition, multiplication, kernel, image, injective, surjective, dimension formula
- INVERTIBILITY: bijective, isomorphism, equivalence of n -dimensional spaces over a given field.
- MATRIX OF A LINEAR MAP: definition, linearity, invertibility, change of basis
- POLYNOMIALS: division algorithm, root, factoring over \mathbb{C} and \mathbb{R}
- INVARIANT SUBSPACES: definition, eigenvalue, eigenvector, linear independence of eigenvectors arising from distinct eigenvalues, upper-triangular matrices, diagonal matrices.
- INNER-PRODUCT SPACES: inner products, norms, Pythagorean Theorem, Cauchy-Schwarz and triangle inequality, orthonormal bases, Gram-Schmidt, orthogonal complements, projections, minimization problems, linear functionals, adjoints
- OPERATORS ON INNER-PRODUCT SPACES: self-adjoint, normal, spectral theorem, normal operators on real inner-product spaces, positive operators, isometries, polar and singular-value decompositions
- OPERATORS ON COMPLEX VECTOR SPACES: generalized eigenvectors, nilpotent, characteristic polynomial, algebraic and geometric multiplicity of eigenvalues, Cayley-Hamilton Theorem, decomposition of an operator by subspaces of generalized eigenvectors, square roots, minimal polynomials, Jordan blocks, Jordan canonical form