Review sheet for APSC 174J

Key Definitions:

- system of linear equations
- real vector space
- subspace
- linear combination
- linear span
- linear dependence and independence
- augmented matrix of a linear system
- row-echelon (upper-triangular) form of a matrix or linear system
- bases of a vector space
- components of a vector with respect to a basis
- dimension of a vector space
- linear transformation
- kernel and image of a linear transformation
- injectivity and surjectivity
- diagonal, upper triangular, and identity matrices
- invertibility and inverse of a square matrix
- determinant of a square matrix
- eigenvalues, eigenvectors, and eigenspaces

Key Theorems

- relation of linear span to solving linear systems (Section 4, Theorem 6)
- If S is a basis of V, then every element of V is uniquely expressible as a linear combination of S (Section 6, Theorem 8)
- dimension of a vector space equals the size of any basis. In particular, any generating set of V is at least as large as $\dim(V)$, and any linearly independent set is no larger than $\dim(V)$. (Section 7, Theorems 9-11)
- The kernel of a linear map $L : V \to W$ is a subspace of V and the image is a subspace of W. (Section 8, Theorem 15 and Section 9, Theorem 19)
- rank-nullity theorem (Section 9, Theorem 20)

- relation of matrices and linear transformations to solving linear systems (Section 10, Theorem 23)
- Equivalent conditions: a square matrix A is invertible if and only if its columns are linearly independent (Section 11, Theorem 26), if and only if its determinant is nonzero (Section 11, Theorem 27).

Procedures

- Gaussian elimination
- Check linear dependence/independence of a set of vectors
- Solve a linear system
- Find a basis, or verify that a given set is a basis, of a vector space
- Compute the kernel and image of a linear map or a matrix
- Matrix addition and multiplication
- Compute determinants
- Compute eigenvalues and eigenvectors of a square matrix or linear operator.