The final exam will cover all topics of this course.


2. Linear transformation.

3. Matrix algebra

4. Vector space and subspace.

5. Eigenvalues and eigenvectors.
Note: These are supplementary questions. Please refer to notes, textbook, homework etc. for review of final exam.

Question 1:

Consider the linear system

\[
\begin{bmatrix}
0 & 1 & 1 & a \\
2 & 4 & 1 & b \\
1 & 2 & 1 & c \\
3 & 7 & 4 & d \\
\end{bmatrix}
\]

Find conditions for which the linear system has (1) no solution, (2) a unique solution, (3) infinitely many solutions.
Question 2:

Let $T$ be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ such that

$$
T(\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}) = \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}, \quad T(\begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}) = \begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix}, \quad T(\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}) = \begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix}
$$

(1) Find the standard matrix $A$.

(2) Find the image of $\vec{v} = \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}$ under $T$. Determine if $\vec{v}$ is in Nul $A$.

(3) Determine if $T$ is one-to-one, onto.
Question 3:

Suppose that $\vec{a}, \vec{b}, \vec{c}$ are the columns of a $3 \times 3$ matrix $A$, i.e. $A = [\vec{a} \vec{b} \vec{c}]$. Suppose that

\[ \vec{a} - 2\vec{b} + 3\vec{c} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad 3\vec{a} + \vec{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{b} - 5\vec{c} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \]

Find the inverse of $A$ and $\det(A)$.

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Question 4:

Let $A = \begin{bmatrix} 1 & 5 & -6 \\ -1 & -4 & 4 \\ -2 & -7 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 & -3 \\ 3 & -3 & 3 \\ 2 & 13 & -7 \end{bmatrix}$, find $\det(-2A^3B^T)$. 

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Question 5:

Determine if $W = \{ p \in \mathbb{P}_3 : p(1) = 0 \}$ is a subspace of $\mathbb{P}_3$. If so, find a basis for $W$ and find its dimension.
Question 6:

Let $T : M_{2 \times 2} \to M_{2 \times 2}$ be defined as

$$T(A) = A - A^T$$

(1) Show that $T$ is a linear transformation.

(2) Find the dimensions of Kernel($T$) and Range ($T$).