MATH 112, Winter 2017
Sample Test #1

Warning: This is a past exam I gave. You should not assume that if a certain topic does not appear here it will not appear on our exam.

Problem 1. Determine if the following statements are true or false.

1. The matrix
   \[
   \begin{bmatrix}
   1 & 0 & 0 & 1 & 0 \\
   0 & 0 & 1 & 1 & 0 \\
   0 & 0 & 0 & 0 & 1
   \end{bmatrix}
   \]
   is in reduced row echelon form.

   Answer: ___________

2. Suppose \(\{\vec{u}, \vec{v}, \vec{w}\}\) are vectors in \(\mathbb{R}^3\). If \(\{\vec{u}, \vec{v}\}\), \(\{\vec{v}, \vec{w}\}\) are linearly independent, then \(\{\vec{u}, \vec{v}, \vec{w}\}\) is also linearly independent.

   Answer: ___________

3. The transformation \(T(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}) = \begin{bmatrix} 2x_1 + 1 \\ -x_2 \end{bmatrix}\) is a linear transformation.

   Answer: ___________

Problem 2. Let \(A\) and \([A|\mathbf{b}]\) be the coefficient and augmented matrix of a linear system. Complete the following table.

| size of \(A\) | rank(\(A\)) | rank([\(A|\mathbf{b}\]) | Number of solutions | Number of free variables in the solution |
|-------------|-------------|----------------|--------------------|------------------------------------------|
| 5 \times 4  | 5           | no solution    |                    | N/A                                      |
| 4 \times 3  |              | one solution   |                    |                                         |
| 6 \times 5  | 4           | 4              | infinitely many solutions |                                      |
| 5 \times 7  |              |                |                    | 4                                        |
Problem 3. Solve the linear system $A\vec{x} = \vec{b}$ where

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & 1 & 0 \\ 3 & 1 & 2 & 7 \\ 1 & 5 & -4 & 1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ -18 \end{bmatrix}$$
**Problem 4.** Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by $T(\vec{x}) = A\vec{x}$, where $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$.

1. Find the preimage of the vector $\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, that is, find $\vec{x} \in \mathbb{R}^2$ such that $T(\vec{x}) = \vec{v}$.

2. Sketch the image of the square with vertices at $(0,0), (1,0), (1,1),$ and $(0,1)$ under $T$. 

![Graph showing the image of the square under the transformation]