Math 112, Homework #2

Part I
Please do the following problems from the textbook:

- §1.3: 11, 13, 21, 25
- §1.4: 1, 3, 11, 15, 25

Part II

Question 1: The RREF’s of some linear systems are given below. Find the solutions of each system.

\[
\begin{bmatrix}
1 & 0 & 7 & 0 & -1 \\
0 & 1 & -5 & 2 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix},
\begin{bmatrix}
1 & 0 & 3 \\
0 & 1 & 4 \\
0 & 0 & 1
\end{bmatrix}
\]

Question 2: For the following table, A represents the coefficient matrix of a linear system, and [A|b] the augmented matrix. Find all possible values for each *, and find out how many solutions the system has for each value you find.

| size of A | rank(A) | rank[A|b] |
|-----------|---------|----------|
| 3 × 4     | *       | 2        |
| 4 × 3     | 3       | *        |
| 4 × 3     | *       | 4        |
| 3 × 4     | 3       | *        |

Question 3: Consider the linear system

\[
\begin{align*}
x_1 + x_2 + 2kx_3 &= 0 \\
x_1 + 2x_2 + 6x_3 &= 2 \\
kx_2 + 4x_3 &= 2
\end{align*}
\]

where k is a constant. For what values of k does

(1) the system have a unique solution?
(2) the system have infinitely many solutions?
(3) the system have no solutions?
Question 4: Write down the vector equation, matrix equation or system of linear equations for each of the following:

(1) \[
\begin{bmatrix}
7 & -3 \\
2 & 1 \\
9 & -6 \\
-3 & 2
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
= 
\begin{bmatrix}
1 \\
2 \\
3 \\
4
\end{bmatrix}
\]

(2) \[
x_1 \begin{bmatrix}
1 \\
2 \\
3 \\
4
\end{bmatrix}
+ x_2 \begin{bmatrix}
-1 \\
-3 \\
0 \\
4
\end{bmatrix}
+ x_3 \begin{bmatrix}
5 \\
6 \\
1 \\
2
\end{bmatrix}
= 
\begin{bmatrix}
2 \\
-5 \\
1 \\
3
\end{bmatrix}
\]

(3) \[
\begin{cases}
x_1 - 3x_2 = 4 \\
2x_1 + 5x_2 = 1 \\
x_1 + x_2 = 2
\end{cases}
\]

Question 5: Could a set of two vectors in \( \mathbb{R}^3 \) span all of \( \mathbb{R}^3 \)? Explain. What about \( n \) vectors in \( \mathbb{R}^m \) when \( n \) is less than \( m \)?