Math 112, Homework #7

Part I Please do the following questions from the textbook.

- §3.3: 7, 13, 25, 27, 31

Part II

Question 1: Consider the linear system:
\[
\begin{align*}
  x_1 + x_2 + 3x_3 &= 1 \\
-2x_1 + 2x_2 + x_3 &= 0 \\
  x_2 + x_3 &= 1
\end{align*}
\]

1. Use Cramer’s rule to find the solution of the system.

2. Use the formula \(A^{-1} = \frac{1}{\det A} \text{adj} A\) to find the inverse of the coefficient matrix, and then use this inverse to find the solution of the linear system.

Question 2: (The adjoint matrix).

1. Show that if \(A\) is invertible, then \(\text{adj} A\) is also invertible, and
\[
(\text{adj} A)^{-1} = \frac{1}{\det A} A
\]

(Hint: Given square matrices \(B\) and \(C\), what relations between \(B, C\) would show that \(C\) is the inverse of \(B\)?)

2. Suppose \(A\) is a 4 \times 4 matrix with \(\det A = -3\), find the determinant of \(\text{adj} A\).

Question 3: (Determinant and area).

1. Find the area of the triangle with vertices \((0, 0), (5, 6), (-2, 7)\).

2. Find the area of the quadrilateral with vertices \((0, 0), (2, 3), (7, 1), (5, -2)\).

(Hint: Check that the quadrilateral is actually a parallelogram. Why?)