Math 231, Tutorial #2

1. Find the general solution and use it to determine how solutions behave as $t \to \pm \infty$.

$$(1 + t^2) \frac{dx}{dt} + 4tx = (1 + t^2)^{-2}$$

2. Determine the asymptotic behavior of solutions to the initial value problem with initial condition $-\frac{\pi}{4} \leq x_0 \leq \frac{\pi}{4}$

$$x' = x \sin(x) - \sin(2x), \quad x(0) = x_0$$

3. For the same differential equation as in Problem 2, show that there must be a critical point between $\frac{\pi}{4}$ and $\frac{\pi}{2}$. What is the stability type of this critical point?

4. Consider the parameterized differential equation (This equation occurs in the study of the stability of fluid flow.)

$$x' = \epsilon x - \sigma x^3, \quad \epsilon > 0, \sigma > 0$$

(a) Solve this equation using the substitution $v = x^{-2}$, and write the solution in terms of its initial value $x(0) = x_0$. Use this formula to show that every solution with initial value $x_0 > 0$ converges to an asymptotic value as $t \to +\infty$. Find this asymptotic value.

(b) Find the equilibrium solutions and determine their stability type. Put the information on a phase line.