MTHE 235, Homework #8

Part I
Please do the following problems from the Textbook.
Section 5.2:
Problem 1, 6, 8, 11, 17, 38

Part II

1. Find the general solution of the following system:
\[ x' = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix} x, \quad x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \]

2. Find the general solution of the following system:
\[ x' = \begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix} x, \quad x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \]

3. Consider the fourth order scalar differential equation:
\[ x^{(4)} - x = 0 \]
(a) Find the general solution of \( x^{(4)} - x = 0 \).
(b) Let \( x_1 = x, x_2 = x', x_3 = x'', x_4 = x''' \), rewrite the scalar equation as a system of first order equations:
\[ x' = Ax, \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \]
(c) Use the result in part (a) to find the general solution of the linear system.
(d) Use the eigenvalue method to find the general solution of the linear system.