MATH 112
Test #1
January 29, 2016
Instructor: Yanxia Deng

Name: __________________________

Student ID number: ____________________

**Instructions:** This is a 50-minute exam. There are 4 questions worth a total of 30 points as indicated in the box below. Answer all questions in the space provided. If you need more room, answer on the back of the previous page. Show all your work and explain how you arrived at your answers, unless explicitly told to do otherwise. No textbooks or notes. Only **CASIO FX-991** or **Gold/Blue Sticker** calculators are permitted.

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Problem 1. Determine if the following statements are true or false. You do not need to justify your answer.

1. The matrix
   \[
   \begin{bmatrix}
   1 & 0 & 0 & 1 & 0 \\
   0 & 0 & 1 & 1 & 0 \\
   0 & 0 & 0 & 0 & 1 \\
   \end{bmatrix}
   \]
   is in reduced row echelon form.

   Answer: _____________

2. Suppose \{\vec{u}, \vec{v}, \vec{w}\} are vectors in \(\mathbb{R}^3\). If \{\vec{u}, \vec{v}\}, \{\vec{v}, \vec{w}\} are linearly independent, then \{\vec{u}, \vec{v}, \vec{w}\} is also linearly independent.

   Answer: _____________

3. The transformation \(T(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}) = \begin{bmatrix} 2x_1 + 1 \\ -x_2 \end{bmatrix}\) is a linear transformation.

   Answer: _____________

Problem 2. Fill the following blanks. You do not need to justify your answer.

| Order of \(A\) | \(\text{rank}(A)\) | \(\text{rank}[A|\text{b}]\) | Number of solutions | Number of free variables in the solution |
|----------------|------------------|------------------|-------------------|----------------------------------------|
| 5 × 4          | 5                |                  | no solution       | 0                                      |
| 4 × 3          |                  |                  | one solution      |                                        |
| 6 × 5          | 4                | 4                |                  |                                        |
| 5 × 7          |                  |                  | infinitely many solutions | 4                                      |
Problem 3. Solve the system $A\vec{x} = \vec{b}$ and write the solution in parametric vector form, where

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & 1 & 0 \\ 3 & 1 & 2 & 7 \\ 1 & 5 & -4 & 1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ -18 \end{bmatrix}$$
Problem 4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by $T(\vec{x}) = A\vec{x}$,

where $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$.

(1) Find the preimage of the vector $\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, that is, find $\vec{u} \in \mathbb{R}^2$ such that $T(\vec{u}) = \vec{v}$.

(2) Sketch the image of the square with vertices at $(0, 0), (1, 0), (1, 1),$ and $(0, 1)$.

\[ x_2 \]
\[ x_1 \]