Math 112, Homework #1

Goals:

• Linear system and augmented matrix.
• Elementary row operation.
• Reduced row echelon form (RREF).

Suggested Textbook Readings: Sections §1.1, §1.2.

Question 1: Determine if the following statements are true or false.

(a) A $5 \times 6$ matrix has six rows.
(b) The solution set of a linear system involving variables $x_1, \cdots, x_n$ is a list of numbers $(s_1, \cdots, s_n)$ that makes each equation in the system a true statement when the values $s_1, \cdots, s_n$ are substituted for $x_1, \cdots, x_n$, respectively.
(c) Every elementary row operation is reversible.
(d) Elementary row operations on an augmented matrix never change the solution set of the associated linear system.
(e) Two matrices are row equivalent if they have the same number of rows.
(f) An inconsistent system has more than one solution.
(g) If one row in an echelon form of an augmented matrix is $[0 \ 0 \ 0 \ 5 \mid 0]$, then the associated linear system is inconsistent.
(h) Whenever a linear system has free variables, the solution set contains infinitely many solutions.

Question 2: Find the linear system with the following augmented matrix. State in words the next two elementary row operations that should be performed in the process of solving the system.

$$
\begin{bmatrix}
1 & -6 & 4 & 0 & -1 \\
0 & 2 & -7 & 0 & 4 \\
0 & 0 & 1 & 2 & -3 \\
0 & 0 & 3 & 1 & 6 \\
\end{bmatrix}
$$
Question 3: Are the following two matrices row-equivalent?

\[
\begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & 6
\end{bmatrix},
\begin{bmatrix}
-4 & -1 & -2 \\
5 & 1 & -1
\end{bmatrix}
\]

How about these matrices?

\[
\begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & 6
\end{bmatrix},
\begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & 0
\end{bmatrix}
\]

Question 4: Find the augmented matrix of the following linear system and solve the system.

\[
\begin{align*}
2x_1 - 2x_3 + x_4 &= -6 \\
4x_2 + 3x_3 + 2x_4 &= 1 \\
-x_1 + 6x_2 - x_3 - x_4 &= 12 \\
\end{align*}
\]

Question 5: Find an equation involving \( a, b, c \) that makes this augmented matrix correspond to a consistent system:

\[
\begin{bmatrix}
1 & -4 & 7 & | & a \\
0 & 3 & -5 & | & b \\
-2 & 5 & -9 & | & c
\end{bmatrix}
\]

Question 6: Write down all types of RREF for 3 \( \times \) 2 matrices.

Question 7: Determine which matrices are in RREF.

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1
\end{bmatrix},
\begin{bmatrix}
1 & 1 & 1 & 0 \\
1 & 1 & 0 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix},
\begin{bmatrix}
1 & 1 & 0 & 0
\end{bmatrix}
\]

Question 8: A bag of pennies, nickels, dimes and quarters contains 120 coins with a total weight 240, worth a total of $12.30. If pennies and dimes weigh 1, nickels weigh 2 and quarters 3, how many coins of each type are there?

Question 9: Consider the following grid of wires.
The temperatures at the exterior grid points are held constant at the temperatures shown (in Celsius). The grid is in thermal equilibrium when the temperature at each interior grid point is the average of the temperatures at the adjacent points. For example

\[ T_2 = \frac{T_1 + T_3 + 0 + 200}{4} \]

Find the values of \( T_1, T_2, T_3 \) for which the grid is in thermal equilibrium.