1. Recall that the Fibonacci sequence is defined as follows: \( F_0 = 0, F_1 = 1 \) and, for \( n \geq 2 \), \( F_n = F_{n-1} + F_{n-2} \). Let \( \alpha = \frac{1 + \sqrt{5}}{2} \) and \( \beta = \frac{1 - \sqrt{5}}{2} \). Prove that
\[
F_n = \frac{1}{\sqrt{5}} (\alpha^n - \beta^n).
\]

2. At a social bridge party every couple plays every other couple exactly once. Assume there are no ties.
   (a) If \( n \) couples participate, prove that there is a “best couple” in the following sense:
       A couple \( u \) is “best” if, for every other couple \( v \), either \( u \) beats \( v \) or \( u \) beats a couple that beats \( v \).
   (b) Show by example that there may be more than one best couple.

3. Let \( n \) be a positive integer. Suppose that there are three pegs and on one of them \( n \) rings are stacked, with each ring being smaller in diameter than the one below it. We want to transfer all the rings to another peg according to the following rules:
   (a) only one ring may be moved at a time,
   (b) a ring may be moved to any peg but may never be placed on top of a smaller ring.
   Note that the second rule implies that the final order on the new peg will be the same as their original order on the first peg.
   Prove that the game can be completed in \( 2^n - 1 \) moves but cannot be completed in fewer moves.