1. (a) Let $a$ and $b$ be integers. Prove that $\text{GCD}(a, b)$ divides $\text{GCD}(a + b, a - b)$. Give an example when $\text{GCD}(a, b) \neq \text{GCD}(a + b, a - b)$.

(b) Let $n$ be an integer. Prove that $\text{GCD}(n^2 - n + 1, n + 1)$ equals 1 or 3. Show by examples that both values are possible.

2. (a) Let $a$ and $b$ be integers with $\text{GCD}(a, b) = d$ and let $a = da', b = db'$. Prove that $\text{GCD}(a', b') = 1$.

(b) Let $a$, $b$, and $n$ be integers such that $\text{GCD}(a, n) = \text{GCD}(b, n) = 1$. Prove that $\text{GCD}(ab, n) = 1$.

**Remark.** In Problems 1 and 2 above you cannot use prime factorization!

3. Find the integer solutions of the following equations:

(a) $306x - 657y = 9$;

(b) $306x - 657y = 15$;

(c) $306x - 657y = 135$. 