1. (a) Let $k$ be a positive integer such that $2^k + 1$ is a prime number. Prove that $k = 2^t$ for some non-negative integer $t$.
(b) Let $k > 1$ be an integer such that $2^k - 1$ is a prime number. Prove that $k$ is prime itself.

2. (a) Prove that there are infinitely many primes of the form $4k + 3$.
(b) For every positive integer $n$, show that there are (at least) $n$ consecutive integers, none of which is prime.

3. Let $A = \{a, b, c\}$. Exhibit a relation on $A$ with the stated properties.
   (a) Reflexive, not symmetric, not transitive.
   (b) Symmetric, not reflexive, not transitive.
   (c) Transitive, not reflexive, not symmetric.
   (d) Reflexive and symmetric, not transitive.
   (e) Reflexive and transitive, not symmetric.
   (f) Symmetric and transitive, not reflexive.