1. Write down monic polynomials in $\mathbb{R}[X]$ of least possible degree which have the following complex numbers as roots:
   (a) $-2$ and $5 + 3i$;
   (b) $1 + i$ and $3i$;
   (c) $5$ and $-\sqrt{7}$.

2. Recall that $m(X) = X^4 + X + 1 \in \mathbb{Z}_2[X]$ is irreducible. Let $\mathbb{K}$ denote the field $\mathbb{Z}_2[X]/(m(X))$ and let $\alpha$ denote $[X]_{m(X)} \in \mathbb{K}$.
   (a) For $b_1, b_2, \ldots, b_k \in \mathbb{K}$ prove the identity
       \[(b_1 + b_2 + \ldots + b_k)^2 = b_1^2 + b_2^2 + \ldots + b_k^2).\]
   (b) Find the roots of $m(X)$ in $\mathbb{K}$. (We know that $\alpha \in \mathbb{K}$ is a root of $m(X)$. The problem is to find the rest of the roots or to prove that there are no more roots.)
   (c) Factor $m(X)$ into irreducible polynomials in $\mathbb{K}[X]$.

3. Prove that $\mathbb{Q}[X]/(X^2 - 2)$ and $\mathbb{Q}[X]/(X^2 - 3)$ are non-isomorphic extensions of $\mathbb{Q}$. In other words, you need to show that both $\mathbb{Q}[X]/(X^2 - 2)$ and $\mathbb{Q}[X]/(X^2 - 3)$ are fields that contain $\mathbb{Q}$ and that they are not isomorphic as fields.