1. (a) Consider $\vec{F}: \mathbb{R}^2 \to \mathbb{R}^2$ given by $\vec{F}(x, y) := \sin(x)\vec{i} + (x + y)\vec{j}$. Find the line integral of $\vec{F}$ around the perimeter of the rectangle with corners $(3, 0)$, $(3, 5)$, $(-1, 5)$, and $(-1, 0)$ traversed in that order.

(b) Let $D$ be a region for which Green’s theorem holds. For any two differentiable functions $P(x, y)$ and $Q(x, y)$, prove that
\[
\int_{\partial D} PQ \, dx + PQ \, dy = \int_D \left[ Q \left( \frac{\partial P}{\partial x} - \frac{\partial P}{\partial y} \right) + P \left( \frac{\partial Q}{\partial x} - \frac{\partial Q}{\partial y} \right) \right] \, dA.
\]

2. (a) If $\vec{F}: \mathbb{R}^2 \to \mathbb{R}^2$ is given by $\vec{F}(x, y) := x\vec{i}$, then show that the line integral of vector field $\vec{F}$ around a closed curve in the $xy$-plane, oriented as in Green’s Theorem, measures the area of the region enclosed by the curve.

(b) Calculate the area of the region within the folium of Descartes $x^3 + y^3 = 3xy$; it is parameterized by $\vec{\gamma}: [0, \infty) \to \mathbb{R}^2$ where $\vec{\gamma}(t) = \left( \frac{3t}{1+t^3} \right)\vec{i} + \left( \frac{3t^2}{1+t^3} \right)\vec{j}$.

3. Consider the vector field $\vec{F}: \mathbb{R} \times (0, \infty) \to \mathbb{R}^2$ given by $\vec{F}(x, y) := \frac{x+y^2}{y^3}\vec{i} - \frac{x^2+1}{y^3}\vec{j}$

(a) Determine if $\vec{F}$ is path-independent.

(b) Find the work done by $\vec{F}$ in moving a particle along the curve $y = 1 + x - x^2$ from $(0, 1)$ to $(1, 1)$.