A Comparative Study of Burst-Noise Communication Channel Models*

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Abstract — In a previous work [7], we introduced and studied the properties of a binary communication channel with memory whose additive noise process is generated according to a finite queue. The queue operates in two modes – a uniform mode and a non-uniform mode – resulting in uniform and non-uniform queue-based channels, respectively. In this work, the capacities of the uniform and non-uniform queue-based channels are compared analytically and numerically with the capacity of the Gilbert-Elliott burst-noise channel. We also consider the problem of fitting our queue-based channels to a typical binary modulated correlated Rayleigh fading channel. This is achieved by estimating the parameters of the queue-based channels that best characterize the error sequence generated by the Rayleigh fading channel.

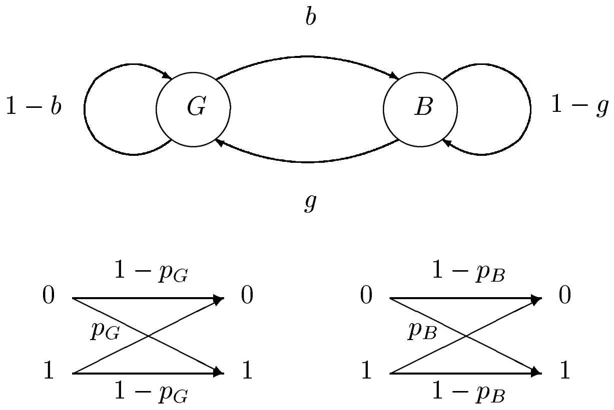

Figure 1: The Gilbert-Elliott channel model.

Keywords: Channel modeling, binary channels with additive burst-noise, error statistics, capacity, correlated Rayleigh fading channel.

1 Introduction

It is well known that the real-world communication channel has memory, often introducing noise distortion in a bursty fashion. In order to design effective communication systems for such a channel, it is important to thoroughly understand its behavior. This is achieved via channel modeling, where the primary objective is to provide a model whose properties are both complex enough to closely capture the real channel statistical characteristics and simple enough to allow mathematically tractable system analysis.

In an attempt to address the above challenging problem, Gilbert initiated in [4] the study of finite-state Markov models for channels with memory by proposing a simple two-state (with one good state and one bad state) model. In the bad state, the channel behaves like a binary symmetric channel (BSC) with a high crossover probability, and in the good state, it behaves like a noiseless BSC. The transitions between the states are governed by a Markov chain. Elliott [2] then suggested a modification to Gilbert's model by introducing a parameter, which denotes the probability of correct reception when the channel is in the good state. The Gilbert-Elliott channel (GEC) is thus a time varying BSC as designed in Fig. 1, where p_G and p_B are the crossover probabilities in the good and bad states, respectively, and g and b are the Markov chain transition probabilities. In a related work, Mushkin and Bar-David introduced a method for calculating tight upper and lower bounds for the capacity of the GEC [5]. Furthermore, Pimentel and Blake expressed the parameters of the GEC as a simple function of the probability of the basis sequences and used the GEC to model a nonfrequency-selective Rician fading channel [6].

In this paper, we extend our investigation of a binary

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communication channel with memory introduced in [7]. The additive noise process of the channel is based on a finite queue with length M. The channel is considered in two cases: a uniform queue-based mode (UQBC) where we experiment on the cells of the queue with equal probability $1/M$, and a non-uniform queue-based mode (NQBC) where we experiment on the cells of the queue with different probabilities, q_1 on cell 1 and $q_l = (1-q_1)/(M-1)$, for cells $l = 2, 3, \ldots M$. It was shown in [7] that the resulting binary noise process $\{Z_i\}$ is a stationary ergodic Mth-order Markov source with the property that each noise sample Z_i depends only on the sum of the previous M noise samples $(Z_{i-1}, Z_{i-2}, \cdots, Z_{i-M})$. The UQBC is completely characterized by three parameters (ε, p and M), while the NQBC is described by four parameters (ε, p, M and q_1) [7]. It was also demonstrated in [7] that for the same bit error rate, correlation coefficient and memory, the UQBC is actually statistically equivalent to the finite-memory Polya contagion channel introduced in [1].

This paper has the following organization. Section 2 compares the capacities of the UQBC and the NQBC with the capacity of the GEC analytically and numerically. The problem of fitting our queue-based channels to a typical binary modulated correlated Rayleigh fading channel (RayFC) is considered in Section 3. We estimate the parameters of the queue-based channel models that best characterize the error sequence generated by the RayFC and compare the queue-based channel models with the GEC model based on two distance measures. A summary is given in Section 4.

2 Capacity Comparisons with the GEC

2.1 UQBC vs GEC

We compare the capacity C_{GEC} of the GEC with the capacity C^M_{UQBC} of the UQBC. C_{GEC} can be computed by evaluating the (asymptotically) tight upper and lower bound introduced in [5]. Unlike the GEC, the capacity (as well as the block transition probability) of the UQBC admits a simple analytical expression [7] in terms of its three parameters (ε, p and M). We first get the following theorem.

Theorem 1 *For $M = 1$, and for the same bit error rate (BER) and noise correlation coefficient (Cor),* $C_{GEC} \geq C^{M=1}_{UQBC}$.

When $M > 1$, only numerical results are obtained. For example, the capacity $C^{M=2}_{UQBC}$ of the UQBC is slightly larger than C_{GEC} for $Cor = 0.1$ (see Figure 2) and $C^{M=2}_{UQBC}$ is less than C_{GEC} for $Cor = 0.9$ (see Figure 3). Thus, for the same BER, the capacity of the UQBC can be either smaller or bigger than that of the GEC, depending on the value of Cor. To see the effect of Cor more clearly, we plot capacity vs Cor in Figure 4 for the GEC and UQBC channels. From Figure 4, we see that $C^{M=2}_{UQBC} < C_{GEC}$ when $Cor > 0.87$ and $C^{M=2}_{UQBC} \geq C_{GEC}$ when $Cor \leq 0.87$.

2.2 NQBC vs GEC

We next compare numerically the capacity $C^{M=2}_{UQBC}$ of the UQBC and the capacity C_{GEC} of the GEC with the capacity $C^{M=2}_{NQBC}$ of the NQBC (with all channels having the same BER and Cor). The results of capacity vs BER are shown in Figures 2 and 3. At a low Cor ($Cor = 0.1$), the three channels have almost identical capacities. For a high Cor ($Cor = 0.9$), the NQBC has the smallest capacity. Additional results are provided in Table 1. Finally, we remark that in the extreme case where the cell probability q_1 tends to one, we get the following two results.

Theorem 2 *For $q_1 \to 1$, for the same BER and Cor,and for any $M = 1, 2, \cdots$,*

$$C^M_{NQBC} \leq C^{M'}_{UQBC}, \quad M' = 1, 2, \cdots. \tag{1}$$

Proof Eq. (1) can be obtained directly by observing that NQBC (with any value of M) converges to the UQBC with memory $M' = 1$ as $q_1 \to 1$ [7] (so $C^M_{NQBC} \to C^{M'=1}_{UQBC}$ as $q_1 \to 1$) and from the fact that memory increases capacity for channels with stationary ergodic Markov additive noise ($C^{M'=1}_{UQBC} \leq C^{M'}_{UQBC}$ for all $M' \geq 1$). □

Theorem 3 *For $q_1 \to 1$, for the same BER and Cor,and for any $M = 1, 2, \cdots$,*

$$C^M_{NQBC} \leq C_{GEC}. \tag{2}$$

Proof Eq. (2) can be obtained directly from Theorem 1 and Theorem 2 (with $M' = 1$). □

The above two theorems are illustrated in Figure 4 where $q_1 = 0.999$. Indeed, we remark that the curves for $C^{M=2}_{NQBC}$ and $C^{M=1}_{UQBC}$ are identical and that $C^{M=2}_{NQBC} \leq C^{M=2}_{UQBC}$ and $C^{M=2}_{NQBC} \leq C_{GEC}$ (exact capacity values for $Cor = 0.1$ and 0.9 are given in Table 1).

3 Modeling of Correlated RayFC

We consider modeling a binary orthogonal frequency-shift keying (FSK) modulated correlated RayFC using our queue-based channels. The same RayFC was studied in [6]. This is achieved by deriving an expression for the probability of an error sequence of length n for the overall RayFC (used with non-coherent demodulation) and choosing the parameters of the queue-based channels that yield the closest statistical behavior. For example, if the UQBC is used, we need to choose the UQBC parameters that minimize the Kullback-Leibler distance (or divergence)

$$D(P^M_{UQBC} \parallel P_{RayFC}) \triangleq \sum_{\boldsymbol{e}_n \in \{0,1\}^n} P^M_{UQBC}(\boldsymbol{e}_n) \log \frac{P^M_{UQBC}(\boldsymbol{e}_n)}{P_{RayFC}(\boldsymbol{e}_n)}, \quad (3)$$

and the variational distance

$$d_v(P^M_{UQBC}(\boldsymbol{e}_n), P_{RayFC}(\boldsymbol{e}_n)) = \sum_{\boldsymbol{e}_n \in \{0,1\}^n} | P^M_{UQBC}(\boldsymbol{e}_n) - P_{RayFC}(\boldsymbol{e}_n) |, \quad (4)$$

where $P^M_{UQBC}(\boldsymbol{e}_n)$ is the block transition probability of the UQBC [7]. $P_{RayFC}(\boldsymbol{e}_n)$ is the probability of an error sequence of length n generated by the correlated RayFC, obtained directly from Eq. (44) [6] (with $K_R = -\infty\, dB$), and is expressed by

$$P_{RayFC}(\boldsymbol{e}_n) = \sum_{l_1=e_1}^{1} \cdots \sum_{l_n=e_n}^{1} \left(\prod_{k=1}^{n} \frac{(-1)^{l_k+e_k}}{l_k+1} \right) \times \frac{1}{\det(\boldsymbol{I} + \frac{E_s}{N_0} \bar{\boldsymbol{C}} * \boldsymbol{F})}, \quad (5)$$

where $\boldsymbol{I}$ is the identity matrix, $\boldsymbol{F}$ is a diagonal matrix defined as $\boldsymbol{F} = diag(\frac{l_1}{l_1+1}, \cdots, \frac{l_n}{l_n+1})$ and $\bar{\boldsymbol{C}}$ is the normalized covariance matrix with entries $\bar{C}_{ij} = J_0(2\pi f_D T|i-j|)$, $1 \leq i,j \leq n$, where $J_0(x) = \sum_{k=0}^{\infty}(-1)^k(\frac{x^k}{2^k k!})^2$ is the zero-order Bessel function of the first kind, f_D is the maximum Doppler frequency experienced by the moving vehicle, T is the symbol interval, E_s is the symbol energy and $N_0/2$ denotes the variance per dimension of the additive Gaussian noise [6].

We consider two cases by choosing the normalized Doppler frequency $f_D T = 0.03$ (which is a representative value for fast fading [6]) and $f_D T = 10^{-4}$ (slow fading) with the average signal-to-noise ratio E_s/N_0 equal to $15\,dB$. For these two cases, the evaluated parameters of the queue-based channel models minimizing the Kullback-Leibler distance and the variational distance when $n = 13$ are given in Tables 2 and 3.

We also compare the queue-based channel models with the GEC model [6] under the same above conditions. We estimate the parameters of the GEC by fitting the correlated RayFC according to the method mentioned in [6]. The exact GEC parameter values are given in Tables 2 and 3. The comparison is based on the Kullback-Leibler and variational distance measures between the probability of error sequences generated by the model and the one generated by the RayFC. The smaller the values of each distance are, the better the model agrees with the RayFC.

The comparison results, shown in Figures 5 and 6, are consistent with respect to the two distance measures. In all cases the GEC model is the best approximation to the RayFC and the UQBC with $M = 1$ is the worst one. This can be explained by the fact that we have limited the memory to $M = 2$ in our queue-based channel models while the GEC (whose noise process is a hidden Markov source) has infinite memory. We expect that the queue-based models will better approximate the RayFC for larger values of M (which necessitate the use of larger values of the block length n). In the case of fast fading the NQBC with $M = 2$ does slightly better than the UQBC with $M = 2$ (see Figures 5 and 6). For slow fading an interesting situation occurs. The curves for the UQBC and the NQBC with $M = 2$ are almost identical (see Figures 7 and 8); this is due to the fact that in this case the NQBC behaves like the UQBC since q_1 is close to $1/2$ (see Table 3).

4 Summary

In this work we extended our investigation of a binary burst-noise channel based on a finite queue. First, we compared the capacities of the UQBC and the NQBC with the capacity of the GEC analytically and numerically. We observed that the capacity of the UQBC (C^M_{UQBC}) is smaller than that of the GEC (C_{GEC}) for the same BER and Cor when memory is 1. In the extreme case where the cell probability $q_1 \to 1$, we observed that the capacity of the NQBC (C^M_{NQBC}) is smaller than that of the UQBC (C^M_{UQBC}) and that of the GEC (C_{GEC}) for the same BER and Cor and for any memory.

Finally, we considered the problem of fitting our queue-based channels to a typical binary modulated correlated RayFC. We estimated the parameters of the queue-based channel models that best characterize the error sequence generated by the RayFC and compared the queue-based channel models with the GEC model based on two distance measures.

In future work, we intend to systematically evaluate the effectiveness of the channel models (including the GEC) for a wide range of signal-to-noise ratios and for various values of fading bandwidth. We are also interested in comparing our proposed queue-based models with the Fritchman channel model [3].

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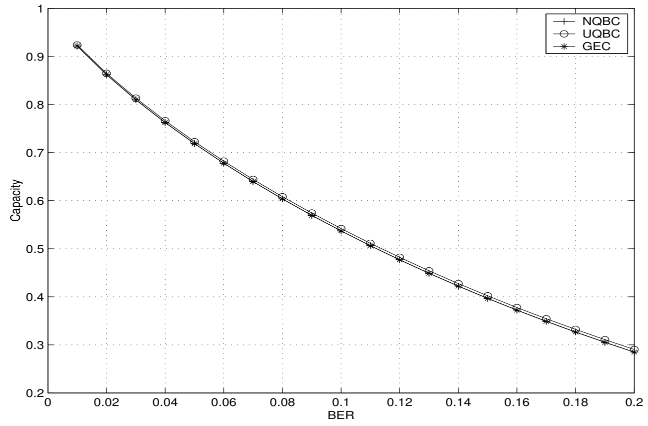

Figure 2: Capacity vs BER for Cor=0.1; M=2 (for UQBC and NQBC), q_1=0.9 (for NQBC), and p_G=0.00002 and p_B=0.92 (for GEC).

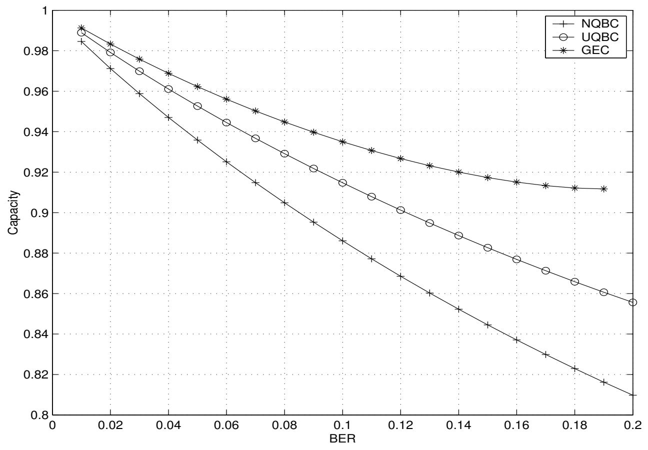

Figure 3: Capacity vs BER for Cor=0.9; M=2 (for UQBC and NQBC), q_1=0.9 (for NQBC), and p_G=0.00002 and p_B=0.92 (for GEC).

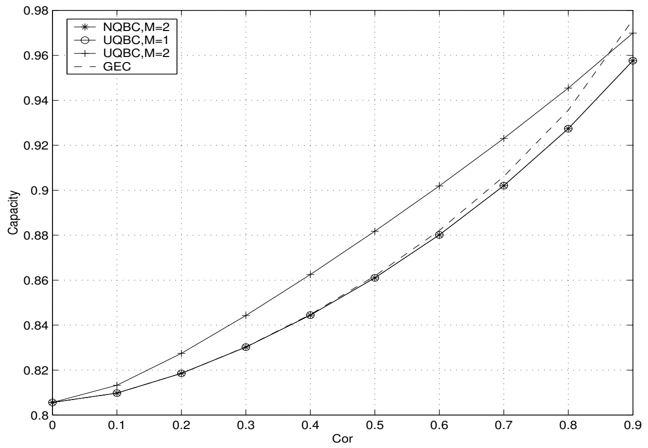

Figure 4: Capacity vs Cor for BER=0.03; q_1=0.999 (for NQBC), and p_G=0.00002 and p_B=0.92 (for GEC).

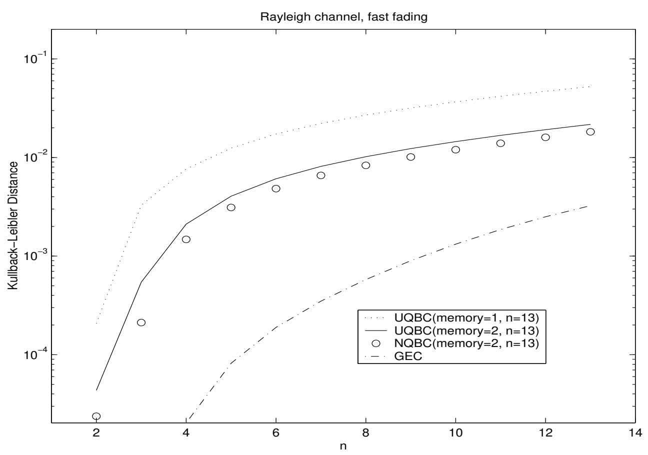

Figure 5: Kullback-Leibler distance for $f_D T = 0.03$.

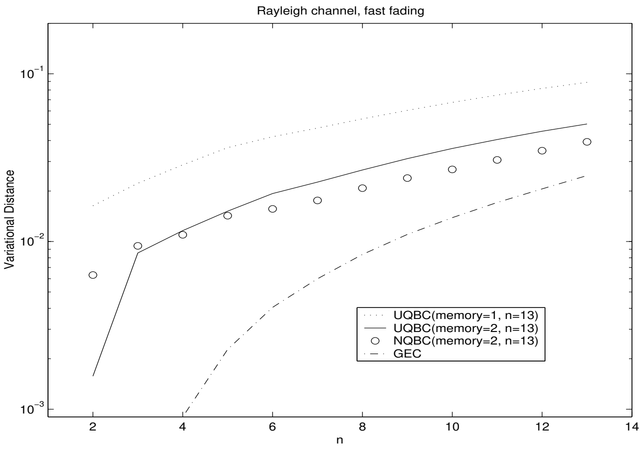

Figure 6: Variational distance for $f_D T = 0.03$.

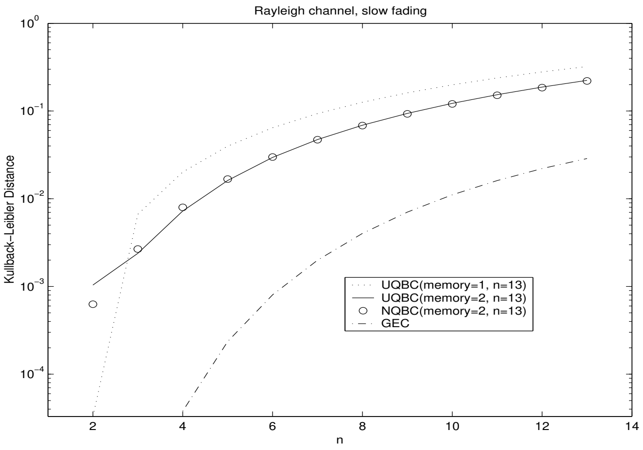

Figure 7: Kullback-Leibler distance for $f_D T = 10^{-4}$.

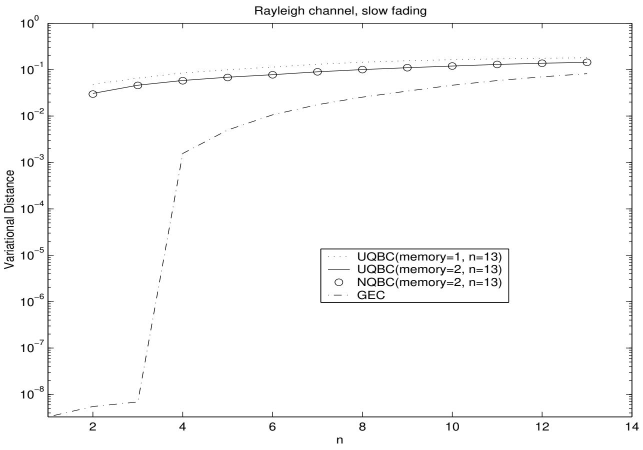

Figure 8: Variational distance for $f_D T = 10^{-4}$.

	Channel models	$M = 1$	$M = 2$	$M = 3$
Cor =0.1	C^M_{UQBC}	0.8098	0.8133	0.8162
	C^M_{NQBC} ($q_1 \to 1$)	0.8098	0.8098 ($q_1 = 0.999$)	0.8098 ($q_1 = 0.999$)
	C^M_{NQBC}	0.8098	0.8123 ($q_1 = 0.55$)	0.8141 ($q_1 = 0.4$)
	C_{GEC}	0.8098		
Cor =0.9	C^M_{UQBC}	0.9576	0.9699	0.9747
	C^M_{NQBC} ($q_1 \to 1$)	0.9576	0.9576 ($q_1 = 0.999$)	0.9576 ($q_1 = 0.999$)
	C^M_{NQBC}	0.9576	0.9681 ($q_1 = 0.55$)	0.9727 ($q_1 = 0.4$)
	C_{GEC}	0.9759		

Table 1: Capacity results for various values of q_1, M and Cor; $BER = 0.03$.

Queue-based models	Minimizing Kullback-Leibler Distance	Minimizing Variational Distance
$UQBC$ $M = 1$	$p = 0.02974$ $\varepsilon = 0.2420$	$p = 0.02974$ $\varepsilon = 0.3449$
$UQBC$ $M = 2$	$p = 0.02974$ $\varepsilon = 0.3135$	$p = 0.02974$ $\varepsilon = 0.3565$
$NQBC$ $M = 2$	$p = 0.02974$ $\varepsilon = 0.3099$ $q_1 = 0.6145$	$p = 0.02974$ $\varepsilon = 0.3528$ $q_1 = 0.6378$
GEC	$b = 0.02338433266528$ $g = 0.27367536719134$ $p_B = 0.34225066080835$ $p_G = 0.00303925823200$	

Table 2: Parameters of queue-based channel models and GEC for Rayleigh fading and $f_D T = 0.03$.

Queue-based models	Minimizing Kullback-Leibler Distance	Minimizing Variational Distance
$UQBC$ $M = 1$	$p = 0.02974$ $\varepsilon = 0.2507$	$p = 0.02974$ $\varepsilon = 0.6542$
$UQBC$ $M = 2$	$p = 0.02974$ $\varepsilon = 0.4905$	$p = 0.02974$ $\varepsilon = 0.6681$
$NQBC$ $M = 2$	$p = 0.02974$ $\varepsilon = 0.5116$ $q_1 = 0.4182$	$p = 0.02974$ $\varepsilon = 0.6689$ $q_1 = 0.4799$
GEC	$b = 0.00000033902448$ $g = 0.00000479118095$ $p_B = 0.33925727777651$ $p_G = 0.00784038925804$	

Table 3: Parameters of queue-based channel models and GEC for Rayleigh fading and $f_D T = 10^{-4}$.