Soft-Decision COVQ Design for Space-Time Orthogonal Block Coded Channels*

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Abstract — **We introduce a soft-decision decoding channel-optimized vector quantizer (COVQ) to transmit analog sources over space-time orthogonal block (STOB) coded channels with binary phase-shift keying (BPSK) modulation. We propose a linear combining method to utilize the signals of all receive antennas. The combiner has a very simple structure when optimized under the maximum signal-to-noise ratio criterion. Furthermore, we note that the concatenation of the system blocks between the COVQ encoder and decoder is equivalent to a discrete memoryless channel (DMC) with a closed-form transition distribution expression. For a COVQ of dimension 2 and rate 1 bps, signal-to-distortion ratio (SDR) gains up of to 0.4 dB with soft-decoding are observed over hard-decoding, translating into up to 2 dB benefits in channel signal-to-noise ratio (CSNR). Performance comparisons with traditional coding systems are also provided.**

I. Introduction

Space-time orthogonal block coding [3, 13] was recently developed to improve the error performance of wireless communication systems. Like many other error protection schemes that are designed in the spirit of Shannon's separation theorem [12], space-time codes are designed to operate on uniform independent and identically distributed (i.i.d.) bit-streams. However, Shannon's separation theorem does not take into consideration constraints on system complexity and delay. As real-world communication systems are constrained, systems with independent source and channel codes, known as tandem systems, may have inferior performance compared with those which perform source and channel coding jointly. Joint source-channel coding may be implemented in various ways. When the input to the space-time encoder is a non-uniform binary sequence, maximum *a posteriori* (MAP) detection may be applied to enhance detection and improve system performance. For single transmit and receive antenna systems, joint source-channel coding via MAP detection is studied, for example, in [2, 11]. For STOB coded channels, this problem is considered in [5], where a closed-form expression for the pairwise error probability (PEP) of symbols which undergo STOB coding and MAP detection is derived and significant improvements are shown over maximum likelihood (ML) detection.

In this work, we consider the transmission of continuous-alphabet (analog) sources over STOB coded channels. We employ COVQ, another joint source channel coding method, for compressing the source while rendering it robust against channel errors. COVQ design was originally studied in [6, 8] for arbitrary discrete single-input single-output (SISO) noisy channels. In [1] and [9], COVQ with soft-decision decoding was implemented for SISO channels with Rayleigh flat fading, and white and colored additive Gaussian noise and ISI, respectively. One main task in COVQ design is to find the channel transition probabilities for the quantizer encoder indices. For channels with multiple receive antennas, another important task consists of properly processing the received signals from the different antennas. We address this by proposing to perform space-time soft-decoding followed by linear combining at the receiver. In our system, the combiner output signal is quantized through an appropriately designed uniform scalar quantizer (as in [1, 9]) with rate q bps. The concatenation of the space-time encoder, the MIMO channel, the space-time soft-decoder, the combiner, and the uniform scalar quantizer is shown to be equivalent to a DMC used kr times, where k and r are the quantizer dimension and rate, respectively. We numerically select the step size of the uniform quantizer, so that the capacity of the equivalent DMC is maximized for each value of CSNR. We show that the transition probabilities of this "equivalent DMC" can be expressed in terms of the symbol PEP of the ML-decoded STOB coded channels. Hence, these probabilities can be determined using the results of [4]. Finally, we design a soft-decision decoding COVQ for the equivalent DMC using the modified generalized Lloyd algorithm (GLA) and evaluate its performance.

II. System Components

This section describes the system elements in detail. The system block diagram is shown in Figure 1.

A. The MIMO Channel

The communication system considered here employs L_T transmit and L_R receive antennas. The baseband constellation points are $c_1 = 1$ and $c_2 = -1$. The channel is assumed to be Rayleigh flat fading, so that the path gain from transmit antenna i to receive antenna j, denoted by H_{ji}, has a unit-variance i.i.d. Rayleigh distribution. We assume that the receiver has perfect knowledge of the path gains. It is also assumed that the channel is quasi-static, meaning that the path gains remain constant during a codeword transmission, but vary in an i.i.d. fashion among codeword intervals. The additive noise at receive antenna j and symbol interval t, N_t^j, is assumed to have a zero-mean unit-variance Gaussian distribution, denoted

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by $\mathcal{N}(0,1)$. Based on the above, for a CSNR of γ_s, the signal at receive antenna j at symbol interval t can be written as $R_t^j = \sqrt{\frac{\gamma_s}{L_T}}\sum_{i=1}^{L_T} H_{ji}s_t^i + N_t^j$, where $\sqrt{\frac{\gamma_s}{L_T}}\{s_t^i\}_{i=1}^{L_T}$ are simultaneously transmitted. In matrix form, we have

$$\boldsymbol{r}_t = \sqrt{\frac{\gamma_s}{L_T}}\boldsymbol{H}\boldsymbol{s}_t + \boldsymbol{n}_t, \tag{1}$$

where $\boldsymbol{r}_t = (R_t^1, ..., R_t^{L_R})^T$, $\boldsymbol{s}_t = (s_t^1, ..., s_t^{L_T})^T$, $\boldsymbol{n}_t = (N_t^1, ..., N_t^{L_R})^T$, $\boldsymbol{H} = \{H_{ji}\}$, and ^{T} denotes transposition.

B. Space-Time Orthogonal Block Codes

In the case of STOB codes with a codeword length of T_W symbol intervals, (1) can be written as

$$\boldsymbol{r}^j = \sqrt{\frac{\gamma_s}{L_T}}\tilde{\boldsymbol{H}}^j\boldsymbol{c} + \boldsymbol{n}^j \qquad j = 1, \cdots, L_R, \tag{2}$$

where $\boldsymbol{c}$ indicates the $\tau \times 1$ vector of transmitted symbols and $\tilde{\boldsymbol{H}}^j$ is derived from the j^{th} row of $\boldsymbol{H}$ via negation of some of its entries. The matrix $\tilde{\boldsymbol{H}}^j$ has orthogonal columns, i.e., $\tilde{\boldsymbol{H}}^{jT}\tilde{\boldsymbol{H}}^j = g\bar{H}_j\mathbf{I}_\tau$, where $\mathbf{I}_n$ is the $n \times n$ identity matrix, $\bar{H}_j = \sum_i H_{ji}^2$, and g is the inverse of the code rate. Equation(2) can be multiplied from the left by $\tilde{\boldsymbol{H}}^{jT}$ to yield the following at the output of the space-time soft-decoder:

$$\tilde{\boldsymbol{r}}^j \triangleq \tilde{\boldsymbol{H}}^{jT}\boldsymbol{r}^j = g'\bar{H}_j\mathbf{c} + \tilde{\boldsymbol{n}}^j,$$

where $\tilde{\boldsymbol{n}}^j \triangleq \tilde{\boldsymbol{H}}^{j\dagger}\boldsymbol{n}^j$ and $g' = g\sqrt{\frac{\gamma_s}{L_T}}$. Note that each entry of $\tilde{\boldsymbol{r}}^j$ is associated with only *one* symbol. It is not hard to show that

$$\tilde{N}_t^j \sim \text{i.i.d. } \mathcal{N}(0, g\bar{H}_j), \quad j = 1, ..., L_R, t = 1, ..., \tau. \tag{3}$$

It follows that symbol i can be detected by only considering the i^{th} entry of the vectors $\tilde{\boldsymbol{r}}^j$, $1 \le j \le L_R$. For our application, this means that the bits corresponding to a VQ index can be detected independently.

It is shown in [4] that the PEP of ML decoded STOB coded symbols equals

$$P(c_i \to c_j) = \mathrm{E}_{\bar{H}}\left\{Q\left(\delta_{ij}\sqrt{\bar{H}}\right)\right\}$$
$$\frac{1}{2}\left(1 - \frac{\delta_{ij}}{\sqrt{2+\delta_{ij}^2}}\sum_{k=0}^{L_T L_R - 1}\binom{2k}{k}\frac{1}{(2\delta_{ij}^2+4)^k}\right), \tag{4}$$

where $Q(\cdot)$ is the Gaussian error function, $\bar{H} = \sum_{j=1}^{L_R}\bar{H}_j = \sum_{i=1}^{L_T}\sum_{j=1}^{L_R}H_{ji}^2$ and $\delta_{ij} = \sqrt{\frac{g\gamma_s}{2L_T}}|c_i - c_j|$. For BPSK modulation, $\delta_{12} = \delta_{21} \triangleq \delta = \sqrt{\frac{2g\gamma_s}{L_T}}$. For future use, we set

$$\Lambda(\delta) \triangleq \mathrm{E}_{\bar{H}}\left\{Q\left(\delta\sqrt{\bar{H}}\right)\right\}. \tag{5}$$

C. Soft-Decision Decoding and the Equivalent DMC

C.1: Linear Combining at the Receiver

We apply linear combining, which is a suboptimal method, to use the space-time soft-decoded signal $(\tilde{R}_i^j)$ of all receive antennas. This problem is a variation of the classical maximum ratio combining (MRC) set-up [10], since the space-time soft-decoded signals have different noise variances (see (3)). Letting $\tilde{\rho}_i^j \triangleq \frac{\tilde{R}_i^j}{g'\bar{H}_j}$, we can write the output of the linear combiner as

$$\tilde{\rho}_i = \sum_{j=1}^{L_R}\alpha_j\frac{\tilde{R}_i^j}{g'\bar{H}_j} = \sum_{j=1}^{L_R}\alpha_j(c_i + \tilde{\nu}_i^j). \tag{6}$$

where c_i is the i^{th} transmitted symbol. From (3), we know that the distribution of $\tilde{\nu}_i^j$ – the noise component of $\tilde{\rho}_i^j$ – is $\mathcal{N}(0, \frac{g}{g'^2\bar{H}_j})$. Therefore, the SNR at the combiner is

$$\mathrm{SNR}_{\mathrm{CO}} = \frac{\left(\sum_{j=1}^{L_R}\alpha_j\right)^2}{\sum_{j=1}^{L_R}\frac{g}{g'^2\bar{H}_j}\alpha_j^2}. \tag{7}$$

In linear combining, the objective is then to choose the weights $\{\alpha_j\}_{j=1}^{L_R}$ so that $\mathrm{SNR}_{\mathrm{CO}}$ is maximized. In order to average the received signals while keeping the received signal power constant, we constrain the weights so that $\sum_{j=1}^{L_R}\alpha_j = 1$. Therefore, the problem would be to minimize the denominator of (7), i.e., finding

$$\min\sum_{j=1}^{L_R}\frac{g}{g'^2\bar{H}_j}\alpha_j^2,$$

subject to $\sum_{j=1}^{L_R}\alpha_j = 1$. Solving the Lagrangian

$$D = \sum_j\frac{g}{g'^2\bar{H}_j}\alpha_j^2 + \lambda\sum_j\alpha_j,$$

we have $\lambda = \frac{-2g}{g'^2\bar{H}}$, and $\alpha_j = \frac{\bar{H}_j}{\bar{H}}$. Therefore, the output of the combiner can be written from (6) as

$$\tilde{\rho}_i = c_i + \tilde{\nu}_i, \tag{8}$$

where $\tilde{\nu}_i = \sum_{j=1}^{L_R}\tilde{\nu}_i^j$. It is easy to verify that

$$\tilde{\nu}_i \sim \mathcal{N}\left(0, \frac{L_T}{g\gamma_s\bar{H}}\right). \tag{9}$$

C.2: The Uniform Quantizer

The linear combiner output, $\tilde{\rho}_i$, is fed into a "uniform" quantizer. Let us indicate the decision levels of this quantizer by $\{z_k\}_{k=-1}^{N-1}$ and its codepoints by $\{w_k\}_{k=0}^{N-1}$, where $N = 2^q$ is the number of demodulation codewords. As the support of $\tilde{\rho}_i$ is the real axis, the quantizer should have two unbounded decision regions. The decision regions of the uniform quantizer are given by

$$z_k = \begin{cases} -\infty, & \text{if } k = -1 \\ (k + 1 - N/2)\Delta, & \text{if } k = 0, \cdots, N-2 \\ +\infty, & \text{if } k = N-1, \end{cases}$$

and the quantization rule $f(\cdot)$ is simply

$$f(\tilde{\rho}) = k, \quad \text{if } \tilde{\rho} \in (z_{k-1}, z_k), \quad k = 0, \cdots, N-1.$$

C.3: The Equivalent DMC

For COVQ design, we need to derive the transition probabilities of the 2^{kr}-input 2^{qkr}-output discrete channel, which is equivalent to the space-time encoder, the MIMO channel, the space-time soft-decoder, the combiner, and the uniform quantizer. As detection of bits which correspond to each quantizer index is decoupled via the use of the space-time soft-decoder, combiner, and uniform quantizer, transmission and decoding of VQ indices are independent of one another, and the discrete channel is equivalent to a binary-input 2^q-output DMC used kr times. We shall refer to this discrete channel as the "equivalent DMC".

The required set of the transition probabilities are $P(w_k|c_1)$ and $P(w_k|c_2)$ for all w_k, where c_1 and c_2 are the BPSK constellation points which, without loss of generality, are assumed to correspond to 1 and 0, respectively. Decision is made in favor of the k^{th} codepoint if the output of the linear combiner falls into the (z_k, z_{k+1}) interval of size Δ. Using (8) and (9) we have

$$\begin{aligned} P(w_k|c_1, \boldsymbol{H}) &= P(z_k \leq 1 + \tilde{\nu}_i < z_{k+1}) \\ &= Q\left(\delta(1 - z_{k+1})\sqrt{\bar{H}}\right) - Q\left(\delta(1 - z_k)\sqrt{\bar{H}}\right). \end{aligned}$$

The expectation over $\boldsymbol{H}$ of each of the above Q functions can be determined using (4):

$$P(w_k|c_1) = \Lambda(\delta(1 - z_{k+1})) - \Lambda(\delta(1 - z_k)), \tag{10}$$

where $\Lambda(\cdot)$ is defined in (5). Similarly, we have

$$P(w_k|c_2) = \Lambda(\delta(1 + z_k)) - \Lambda(\delta(1 + z_{k+1})). \tag{11}$$

Note that the DMC transition probability matrix is symmetric in the sense of [7].

Consider a k dimensional quantizer with rate r as shown in Figure 1. Let us denote the natural binary representation of the index of decision region S_i by $\{b_l\}_{l=1}^{kr}$ and that of codevector w_j by $\{m_l\}_{l=1}^{kr}$, where m_l is a binary q-tuple. As the DMC is memoryless, the COVQ index transition probabilities can easily be computed by

$$P(j|i) = \prod_{l=1}^{kr} P\left(w_{m_l}|c_{(2-b_l)}\right). \tag{12}$$

D. The Step-Size Δ

Table I lists the capacity of the DMC derived after quantizing the space-time soft-decoded channel output with q bits. For a given q and CSNR γ_s, we determine the step-size Δ which maximizes the capacity of the DMC via maximizing the mutual information between the DMC input and output. Because the channel transition probability matrix is symmetric, a uniform input distribution can achieve channel capacity [7]. As shown in Figure 2, when the step-size is very small or very large, soft-decoding does not increase channel capacity. It also shows that if CSNR is high, soft-decoding is not very beneficial. With the optimal choice of Δ, though, soft-decoding significantly increases the channel capacity, specially when the receiver noise is strong. For example, at CSNR = -2 dB, there is a 15% benefit in using soft-decoding with $q = 5$ bits. Also note that channel capacity increases less than 1% from $q = 3$ to $q = 5$ even for the worst channel conditions. This shows that typically $q = 3$ would achieve almost the whole gain in soft-decoding.

III. Soft-Decoding COVQ for the Equivalent DMC

The transition probability given in (12) can be used in the modified GLA algorithm to design a soft-decoding COVQ for space-time coded MIMO channels as explained below. Every input k-tuple is encoded at a rate of r bits per sample. Therefore, the input space is partitioned into $N_e = 2^{kr}$ subsets. As we use BPSK modulation, a vector of kr real-valued signals is received for every transmitted index. This vector is softly demodulated at a rate of q bps. Therefore, each k-dimensional source vector is decoded to one of the $N_d = 2^{qkr}$ codevectors. The input space partitioning and the codebook are optimized in an iterative fashion using a number of training vectors as follows.

The nearest neighbor condition: for a fixed codebook and $i = 0, \cdots, N_e - 1$, the optimal partition $\mathcal{P}^* = \{S_i^*\}$ is

$$S_i^* = \left\{\boldsymbol{x} : \sum_{j=0}^{N_d-1} P(j|i) d(\boldsymbol{x}, \boldsymbol{y}_j) \leq \sum_{j=0}^{N_d-1} P(j|l) d(\boldsymbol{x}, \boldsymbol{y}_j), \forall l\right\}$$

where $\boldsymbol{x}$ is a training vector and $d(\boldsymbol{x}, \boldsymbol{y})$ is the Euclidean distance between $\boldsymbol{x}$ and $\boldsymbol{y}$.

The centroid condition: given a partition $\mathcal{P}$, the optimal codebook $\mathcal{C}^* = \{\boldsymbol{y}_j^*\}$ is

$$\boldsymbol{y}_j^* = \frac{\sum_{i=0}^{N_e-1} P(j|i) \sum_{l:\boldsymbol{x}_l \in S_i} \boldsymbol{x}_l}{\sum_{i=0}^{N_e-1} P(j|i)|S_i|}, \qquad j = 0, \cdots, N_d - 1.$$

where $|S_i|$ is the number of the training vectors in S_i.

IV. Numerical Results

We consider a dual-transmit single-receive antenna system using Alamouti's space-time code [3]. Tables II and III demonstrate the results of soft-decoding for zero-mean unit-variance i.i.d. Gaussian and Gauss-Markov sources, respectively. For the sake of comparison, the performance of other traditional coding systems is also provided. The quantizer rate is 1 bps and its dimension is 2. As expected, using more soft-decoding bits increases the SDR. Also, a large gain is observed in using COVQ instead of VQ (which does not exploit the channel statistics). For example, for the Gauss-Markov source and at CSNR = -2 dB, the SDR of the soft-decoding COVQ with $q = 3$ bits is 0.4 dB higher than that of the hard-decoding COVQ and 2 dB higher than the VQ-based system. We also consider a tandem coding scheme consisting of a rate 0.5 bps VQ and a 64-state rate 1/2 convolutional code (denoted by VQ+CC). Interestingly, we note that this system is outperformed by the COVQ for all shown values of CSNR. Except for a limited range of CSNR (1 dB $< \gamma_s <$ 3 dB) and only for the Gauss-Markov source, using only a VQ turns out to be better than using a VQ and convolutional code; this indicates that the performance loss due to the reduced bit rate in source coding is more important than the error protection gain through channel coding. We also observe that the soft-decoding gain is minimal for $q > 3$. This agrees with our capacity results in Table I and Figure 2.

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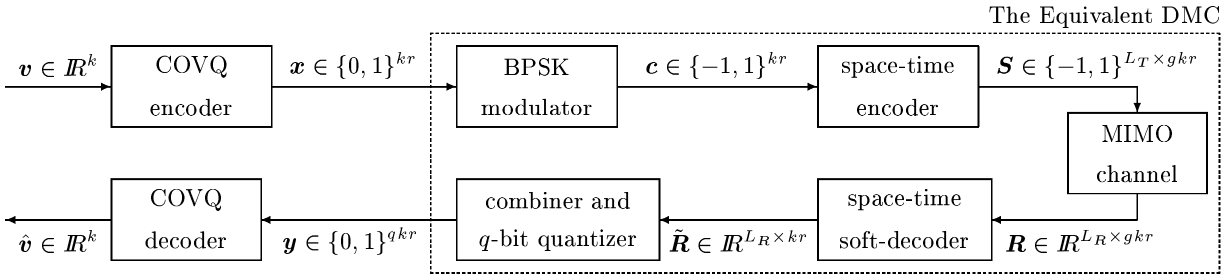

Figure 1: System block diagram.

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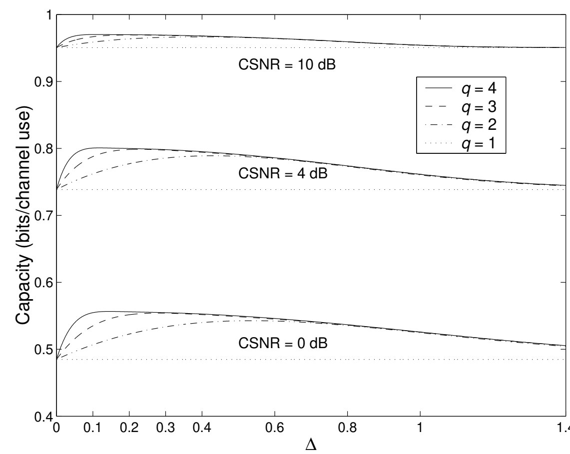

Figure 2: Capacity vs. the step-size of the uniform quantizer.

CSNR (dB)	Capacity (bits/channel use)				
	$q=1$	$q=2$	$q=3$	$q=4$	$q=5$
16.0	0.9945	0.9969	0.9973	0.9974	0.9974
14.0	0.9881	0.9929	0.9937	0.9939	0.9940
12.0	0.9752	0.9842	0.9858	0.9862	0.9864
10.0	0.9506	0.9660	0.9695	0.9702	0.9705
8.0	0.9070	0.9333	0.9381	0.9392	0.9397
6.0	0.8374	0.8760	0.8833	0.8849	0.8855
4.0	0.7386	0.7891	0.7987	0.8009	0.8015
2.0	0.6164	0.6741	0.6852	0.6877	0.6884
0.0	0.4849	0.5427	0.5540	0.5565	0.5572
-2.0	0.3608	0.4142	0.4223	0.4246	0.4252
-4.0	0.2560	0.2974	0.3057	0.3076	0.3081

TABLE I- Capacity (in bits/channel use) of the DMC derived from q-bit soft-decoding of BPSK-modulated space-time coded MIMO channel with $L_T = 2$ and $L_R = 1$.

CSNR (dB)	COVQ			Other Systems	
	$q=1$	$q=2$	$q=3$	VQ	VQ+CC
10	4.155	4.159	4.161	4.155	1.664
8	3.919	3.982	4.003	3.913	1.664
4	2.933	3.198	3.245	2.845	1.642
0	1.615	1.918	1.969	1.228	0.540
-2	1.078	1.323	1.368	0.457	-0.486

TABLE II- Simulated SDR in dB for an i.i.d. $\mathcal{N}(0,1)$ source vector quantized at rate 1 bps and soft-decoded with q bits. Quantization dimension is 2; $L_T = 2$ and $L_R = 1$ (CC: convolutional code).

CSNR (dB)	COVQ			Other Systems	
	$q=1$	$q=2$	$q=3$	VQ	VQ+CC
10	7.301	7.311	7.314	7.273	4.027
8	6.744	6.869	6.938	6.627	4.027
4	4.937	5.316	5.400	4.395	3.966
0	3.014	3.382	3.451	1.758	1.127
-2	2.242	2.563	2.629	0.627	-0.874

TABLE III- Simulated SDR in dB for a zero-mean unit-variance Gauss-Markov source ($\rho = 0.9$) vector quantized at rate 1 bps and soft-decoded with q bits. Quantization dimension is 2; $L_T = 2$ and $L_R = 1$ (CC: convolutional code).