VQ-Based Hybrid Digital–Analog Joint Source–Channel Coding

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I. INTRODUCTION
Consider a system designed for conveying a d-dimensional random source vector, X. A sample, x, from the source is fed to the encoder e, producing an index i = e(x) ∈ {0, . . . , N − 1}, where N = 2d. The L bits of i are then fed to a binary symmetric channel (BSC), resulting in the output j producing a codeword yj from the decoder codebook {yj}N−1j=0. We assume that the BSC corresponds to a Gaussian channel with noise variance σ2 and with binary input in {±1}.

At the transmitter, the index i also chooses a codeword zi from the encoder codebook {zi}N−1i=0, and the residual vector e = x − zi is then formed. This vector is scaled by the constant α and transmitted over a discrete-time analog-amplitude Gaussian channel. (The scaling constant α regulates the transmission power.) The received vector u = α · e + w, where w is zero-mean Gaussian with independent components of variance σ2, is multiplied by a re-scaling constant β and then added to the codeword yj, resulting in an estimate of the transmitted source vector according to

\[ \hat{x} = \beta u + y_j. \]

Hence, the reproduction \( \hat{x} \) is based on information transmitted both via a digital and an analog channel. This is the key principle behind the work of this paper. Related previous work can be found in, e.g., [1, 2].

II. SYSTEM DESIGN AND PERFORMANCE
We will now present optimality criteria for the described HDA system, resulting in a design algorithm striving to minimize the distortion D = E[∥X − \( \hat{X} \)∥2] under a constraint on the transmitted power Pα per channel use in the analog channel. More precisely, the aim of the design is to find an (X), \{zi\}, \{yj\} and \( \beta \) such that D is minimized, under the constraint that \( \alpha \) is chosen such that \( P_\alpha = 1 \) is satisfied at all times.

Optimality for a fixed encoder. Assume that \( \varepsilon(x) \) is known and fixed, and define

\[ \hat{x}(j) \triangleq \mathbb{E}[X|J = j], \quad f_k \triangleq \sum_{i=0}^{N-1} \text{Pr}(I = i|J = j) \text{Pr}(J = k|I = i) \]

and the matrices

\[ \mathbf{Y} \triangleq [y_0 \ldots y_{N-1}], \quad \mathbf{\hat{X}} \triangleq [\hat{x}(0) \ldots \hat{x}(N-1)], \quad \text{and} \quad (\mathbf{F})_{kj} = f_{kj}. \]

Then the optimal encoder and decoder codebooks, \{zi\} and \{yj\}, can be jointly determined, by solving the equation

\[ \mathbf{Y} \cdot (\mathbf{I}_N - \gamma \mathbf{F}) = (1 - \gamma) \mathbf{\hat{X}}, \]

where \( \mathbf{I}_N \) is the N × N unity matrix and \( \gamma \triangleq \alpha \beta \), and then letting \( z_i = m_{0i}(i) \triangleq E[y_j|I = i] \). Furthermore, the optimal \( \beta \) can be found (indeed determined) as \( \beta = C/(1 + \sigma^2) \).

Optimality for a fixed decoder codebook. Now assume that \{yj\} is given, that \{zi\} is chosen as \( z_i = m_{1i}(i) \), and that \( \beta = C/(1 + \sigma^2) \), as above. The optimal encoder then is

\[ \varepsilon(x) = \arg \min \{ (1 - \gamma) \cdot ||x - m_{1i}(i)||^2 + g_i \}, \]

where \( g_i \triangleq E[|y_j|^2|I = i] - ||m_{1i}(i)||^2 \). Based on these results, the system can be (locally) optimized at an assumed channel SNR, 1/\( \sigma^2 \), using an iterative approach similar to the well-known generalized Lloyd algorithm for VQ design.

Motivated by a broadcast scenario, we illustrate below the performance (signal-to-distortion ratio versus SNR) of employing a fixed encoder and an adaptive decoder (adapts to a varying SNR), denoted by FE, AD where \( * \) is the design SNR of the encoder. We also illustrate some benchmark schemes. All systems use a rate of two channel uses per source sample. The source is Gaussian Markov with correlation 0.9.

Dashed lines from above at SNR = 15 dB: The Shannon bound (distortion-rate function evaluated at channel capacity); a purely analog system (transmits each source sample twice, minimum mean-square error receiver); a purely digital tandem system (source-optimized VQ with \( d = 8 \) and \( L = 8 \), rate-1/2 Turbo code with \( (n, k) = (2048, 1024) \) and generators \( (37, 21) \)). Solid lines from above at SNR = 15 dB: A HDA system with source-optimized VQ, and; HDA–FE, AD systems with \( * = 0, 5, 0 \) dB. All HDA systems use \( d = 8 \) and \( L = 8 \).

We observe that the HDA systems outperform the tandem system and the analog system (at high SNRs). In particular, we note the graceful improvement of the HDA systems, as opposed to the leveling-off in performance of the tandem system. We also observe that the performance can be improved at low SNRs using the optimization procedure.

REFERENCES

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